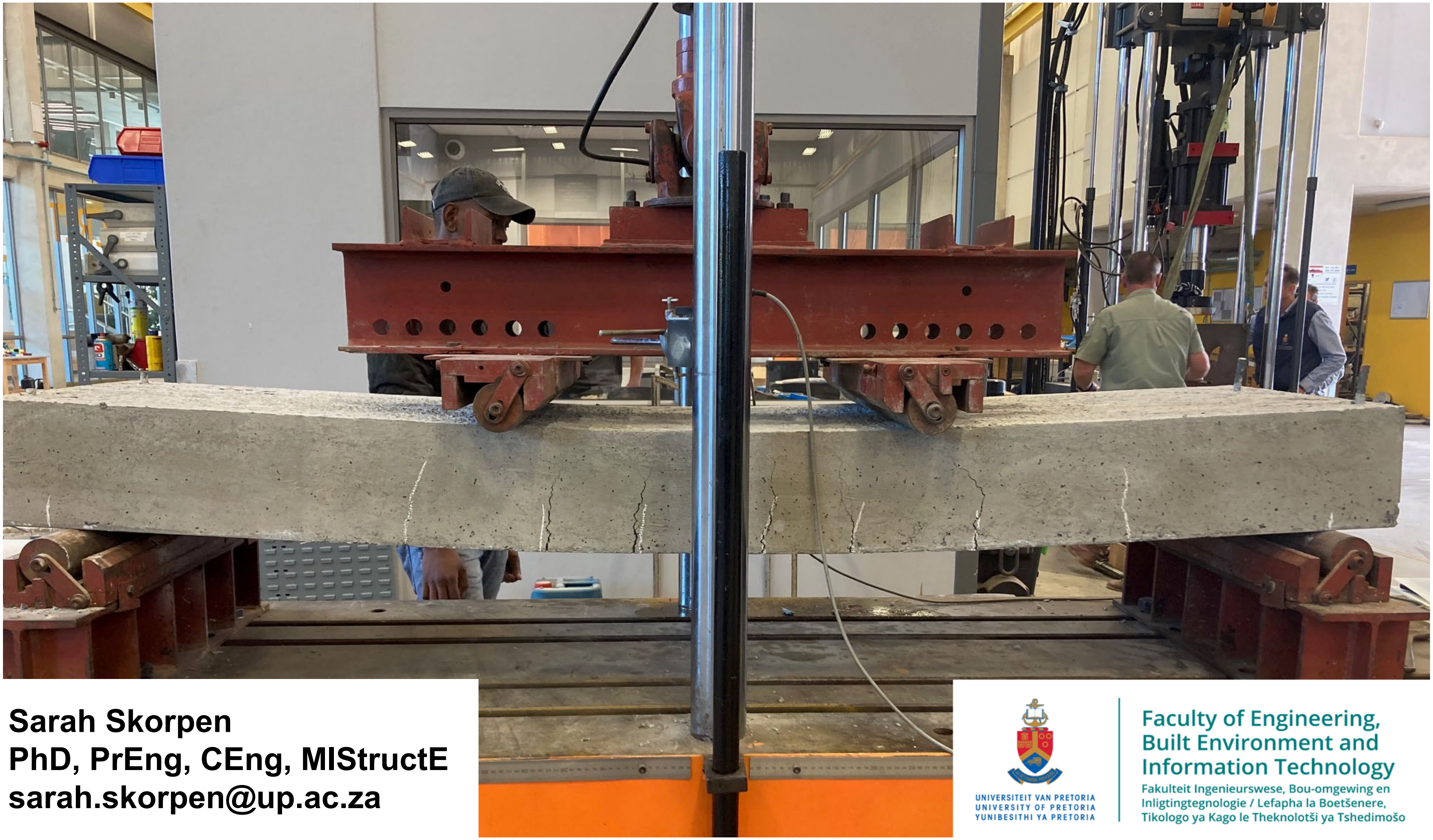
CONCRETEClass - Back to Basics 1:9

Reinforced concrete design: design for flexure with and without axial force



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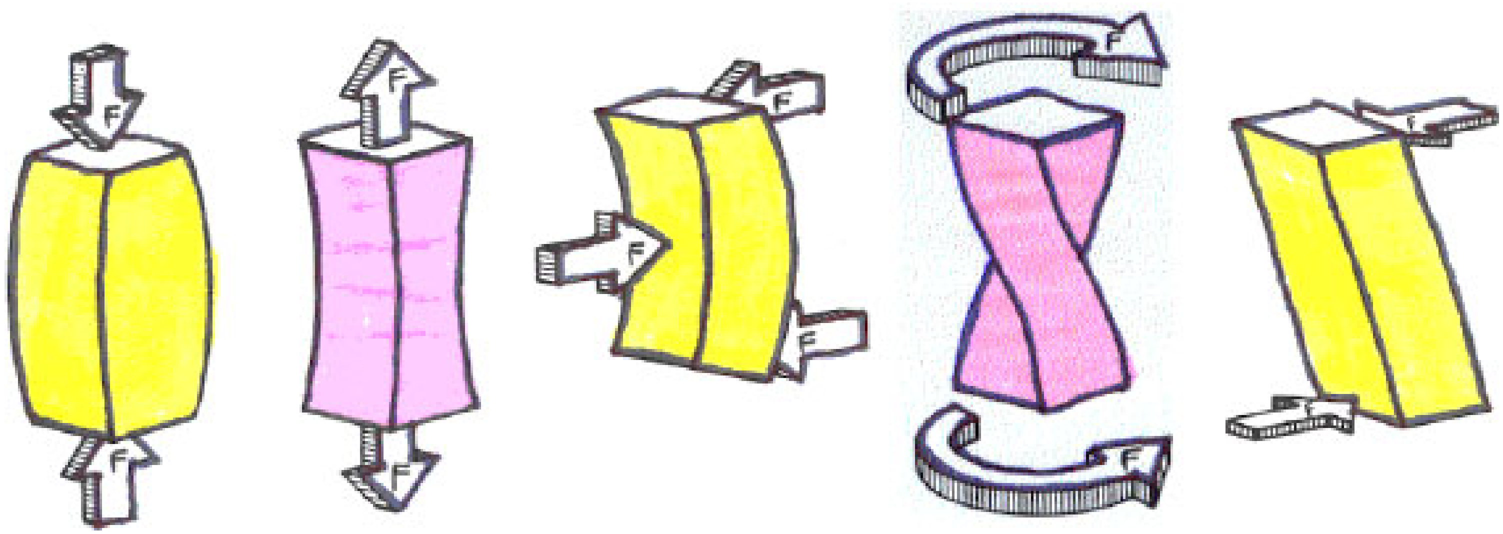
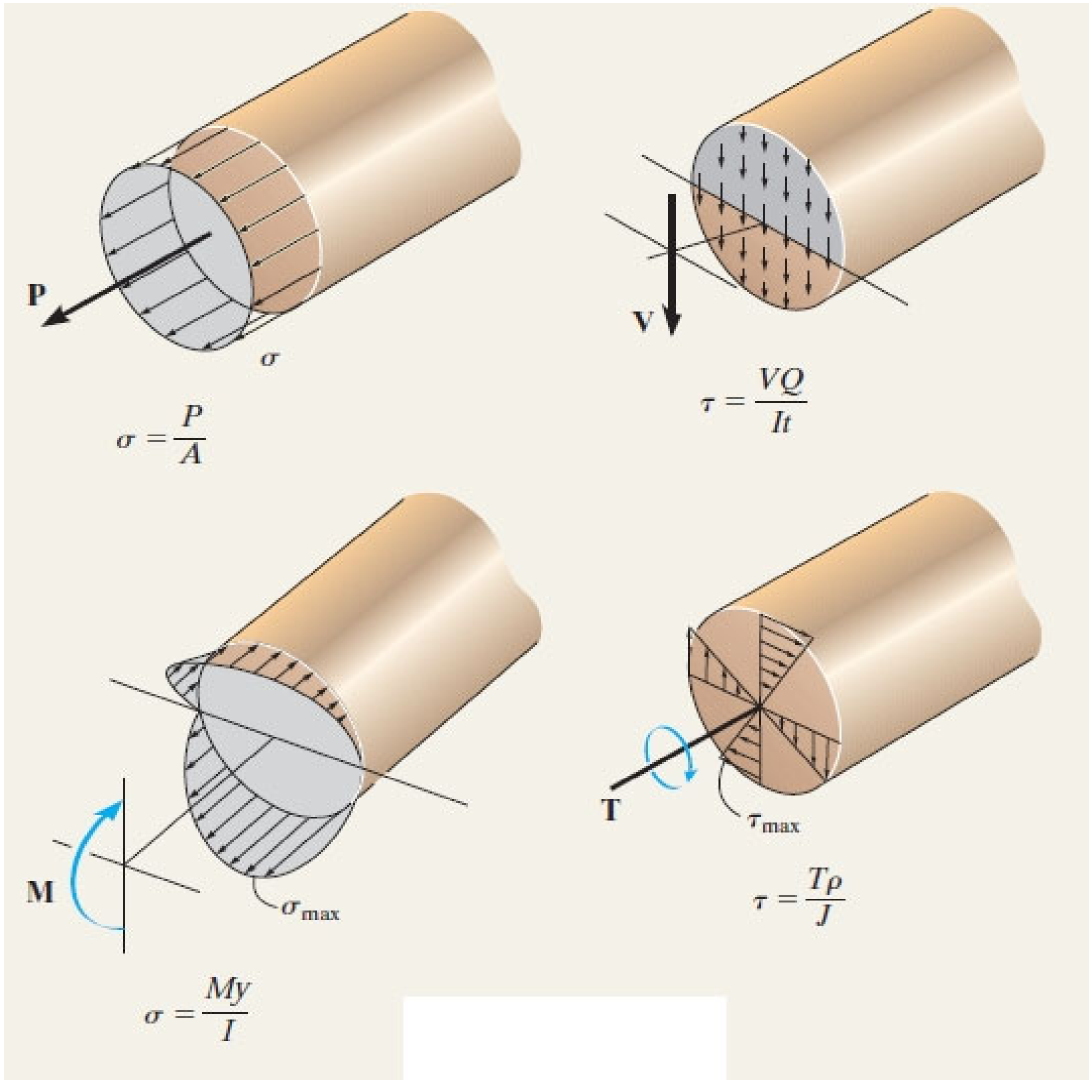
LOAD EFFECTS



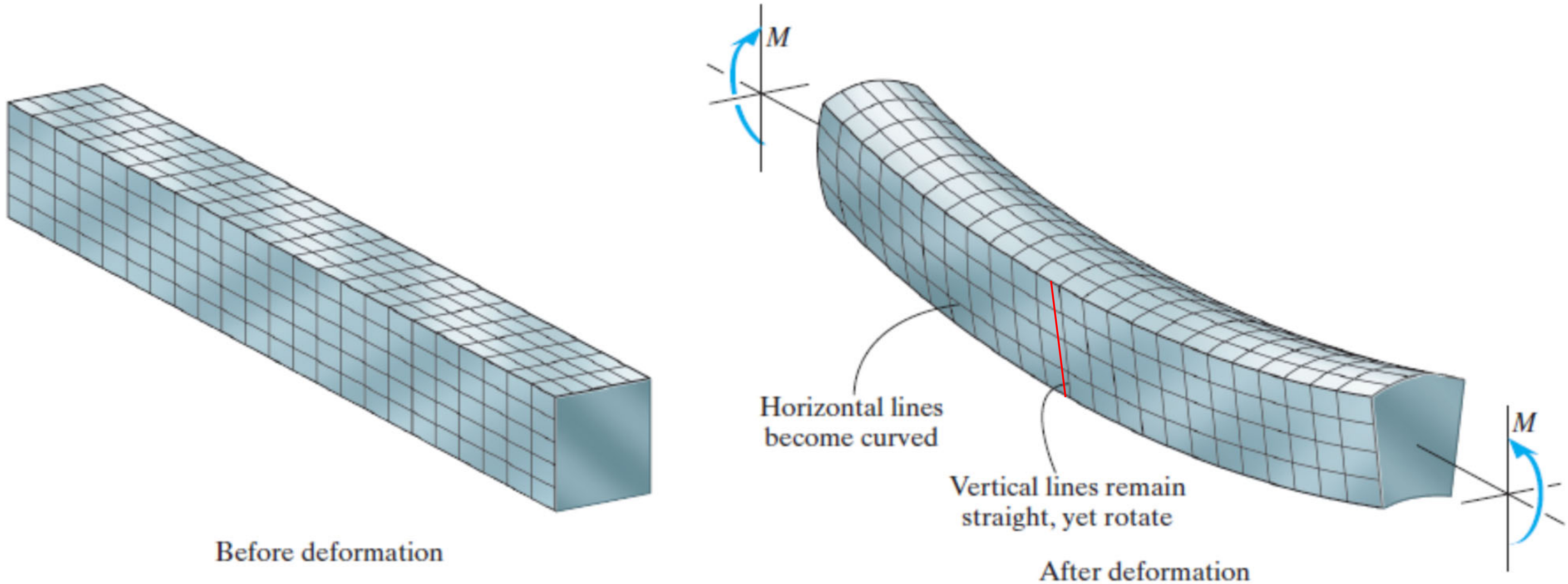
Axial force (N) **Bending (M)** **Torsion (T)** **Shear (V)**

Tension / Compression

STRESS DISTRIBUTION



STRESS AND STRAIN DISTRIBUTION



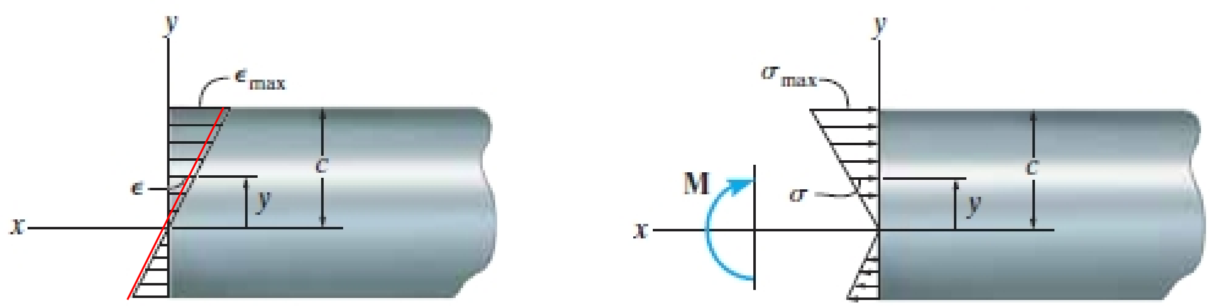
Before deformation

(a)

After deformation

(b)

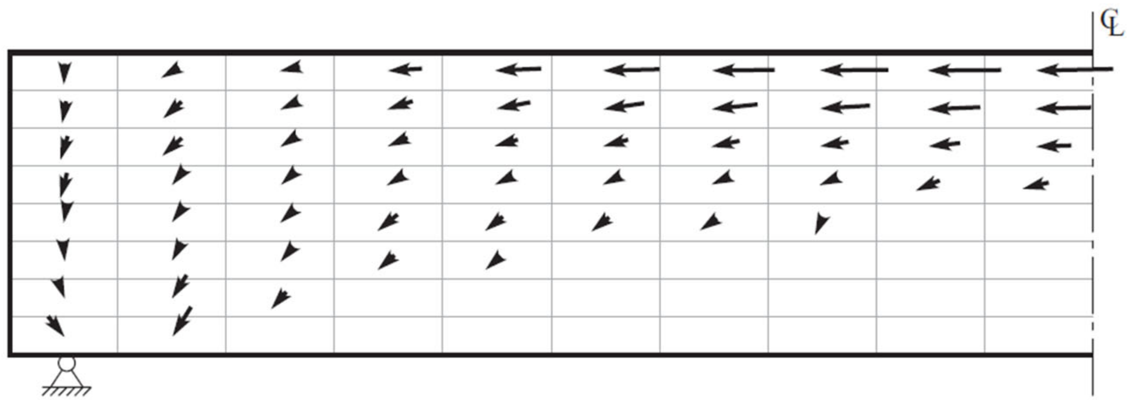
Bending deformation of a straight member



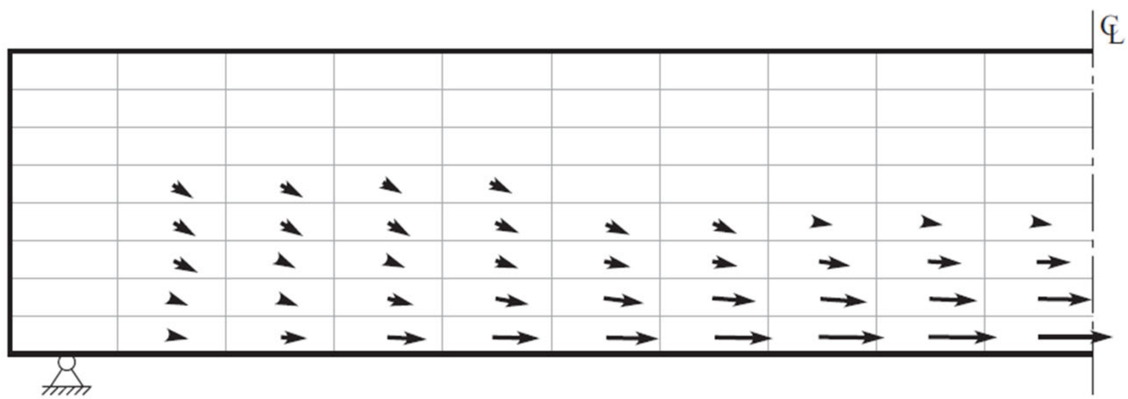
Normal strain variation (profile view)

Bending stress variation (profile view)

PRINCIPAL STRESSES



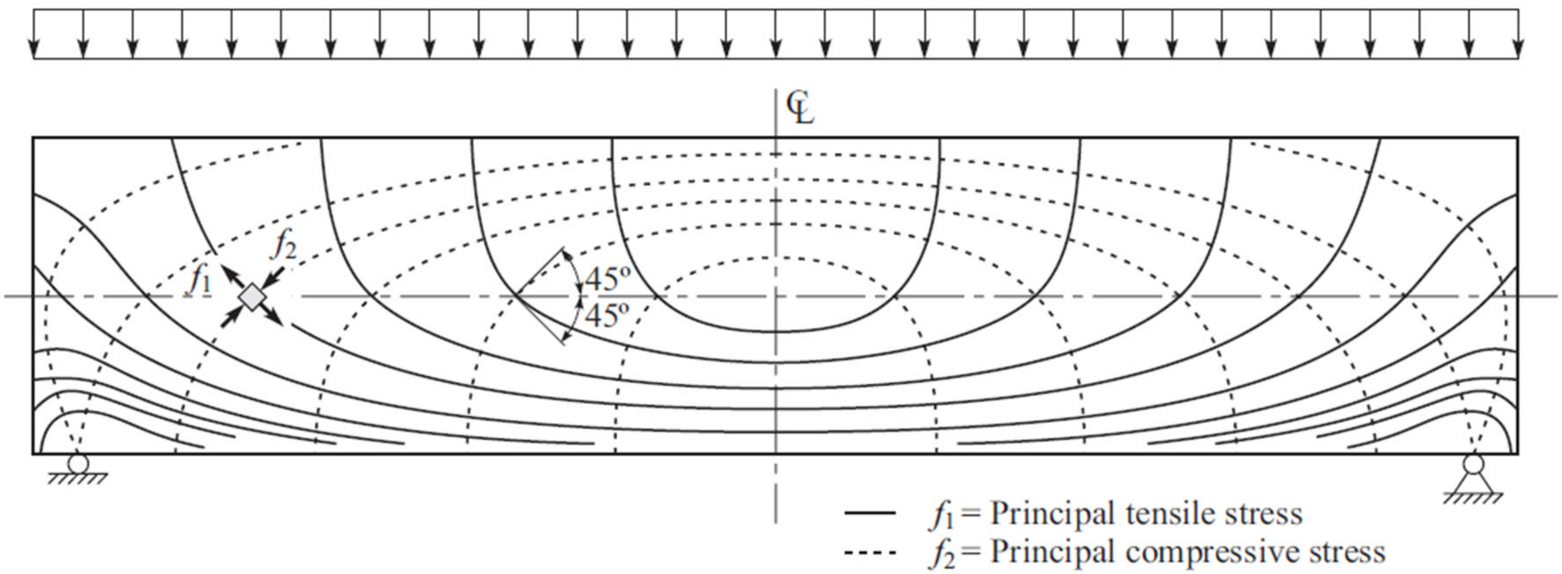
Minimum principal stress (Compression)



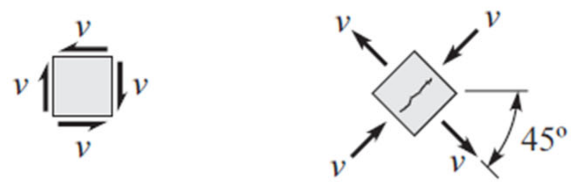
Maximum principal stress (Tension)

Principal stresses in a simply supported beam with UDL (Robberts and Marshall 2009)

STRESS TRAJECTORIES



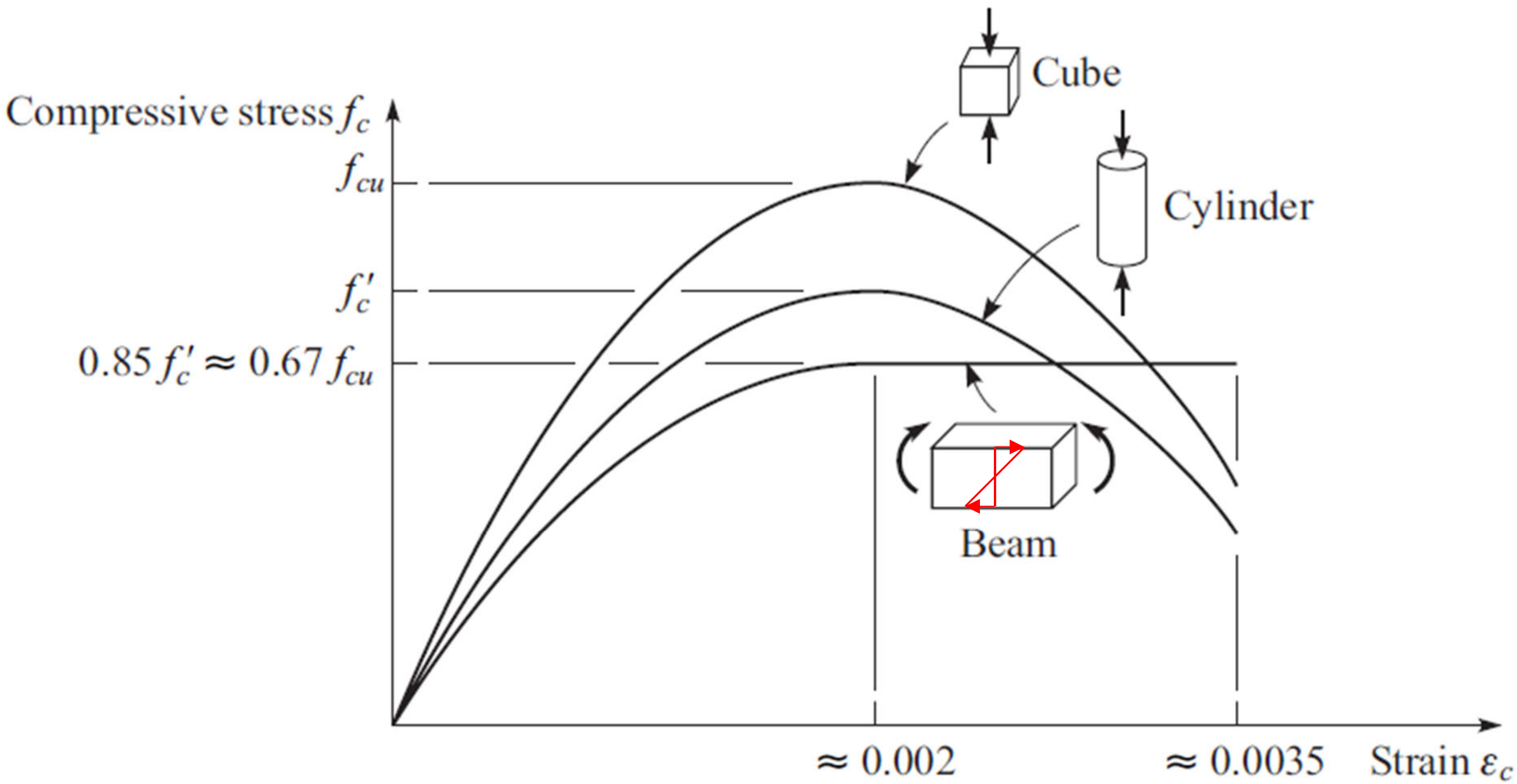
(a) Principal stress trajectories



(b) Stresses at neutral axis

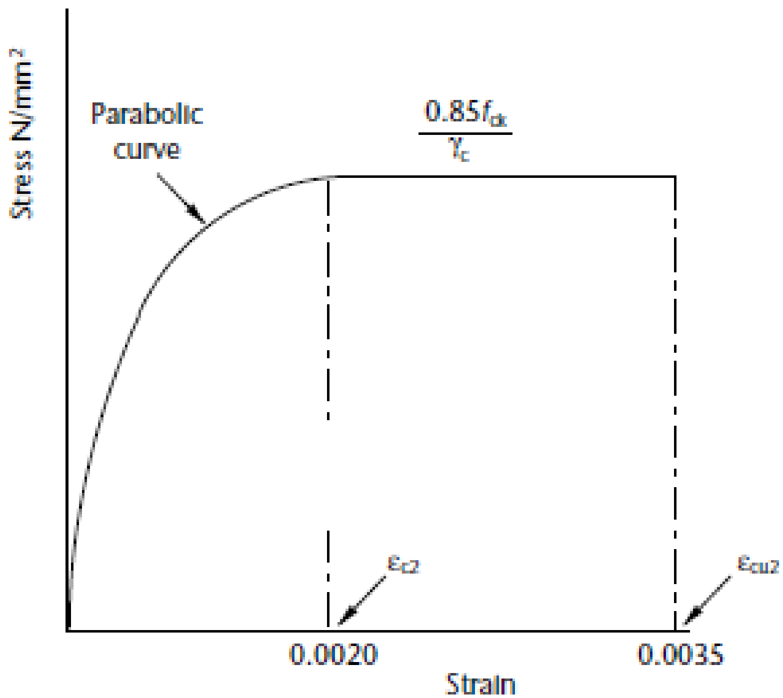
Principal stresses trajectories for a simply supported beam (Robberts and Marshall 2009)

CONCRETE STRESS - STRAIN GRAPH



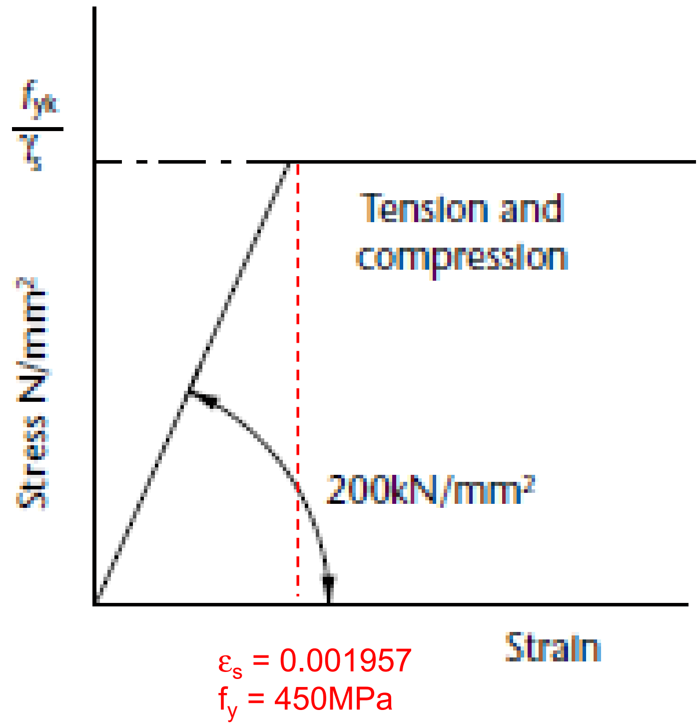
Stress-strain behavior of different concrete test specimens (Robberts and Marshall 2009)

MATERIAL PROPERTIES



Simplified stress-strain relationship for concrete (Mosley et al. 2012)

Concrete is non-linear, not ductile and pre-cracked. Tensile strength is 10-15% of the compressive strength

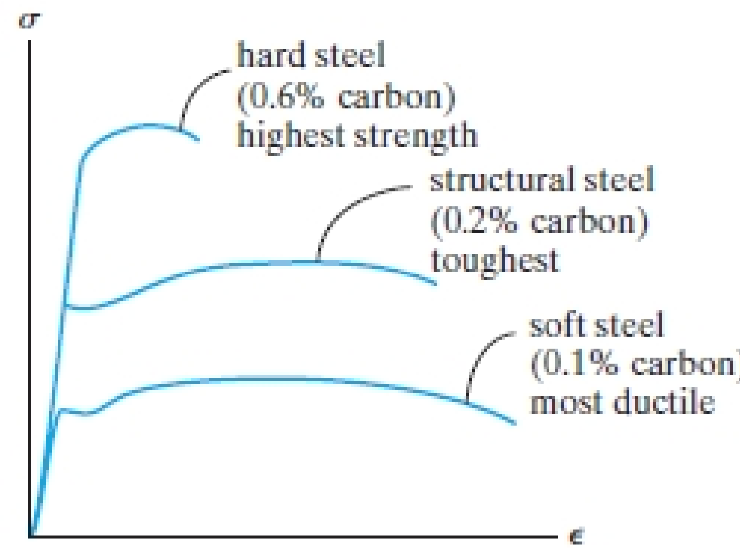


Simplified stress-strain relationship for steel (Mosley et al. 2012)

Steel is linear and ductile

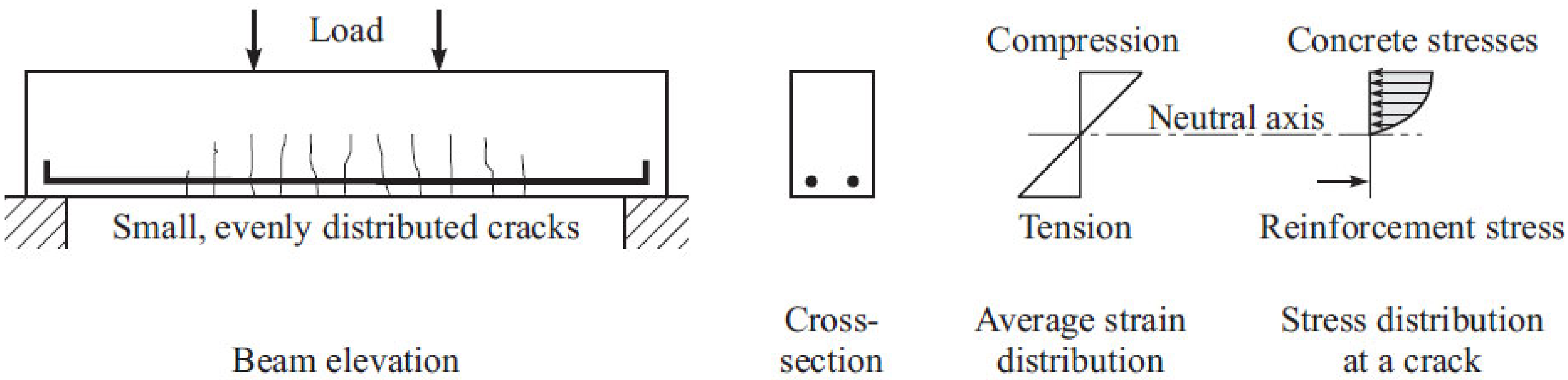
DUCTILITY

Brittle material

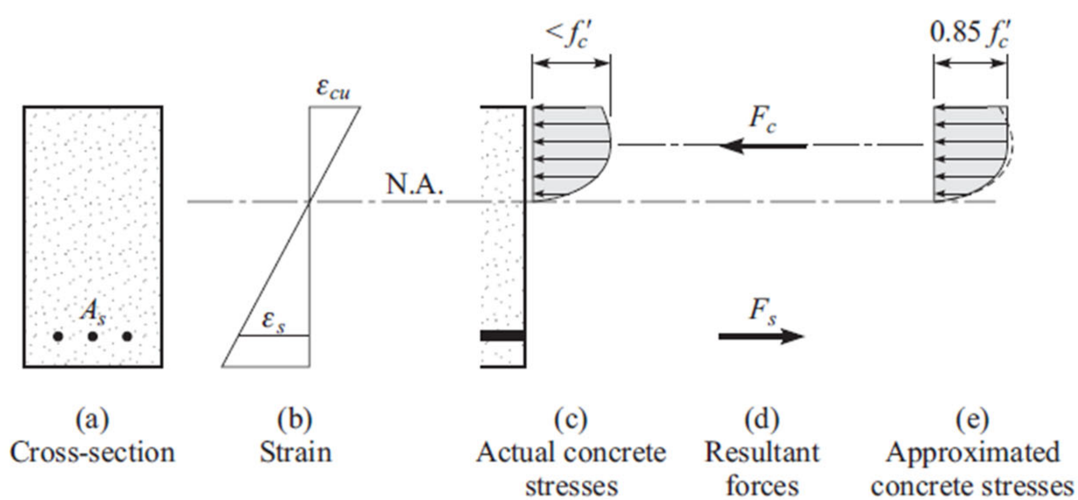


Ductile material - Steel

REINFORCED CONCRETE – A COMPOSITE



Composite action between concrete and reinforcement (Robberts and Marshall 2009)

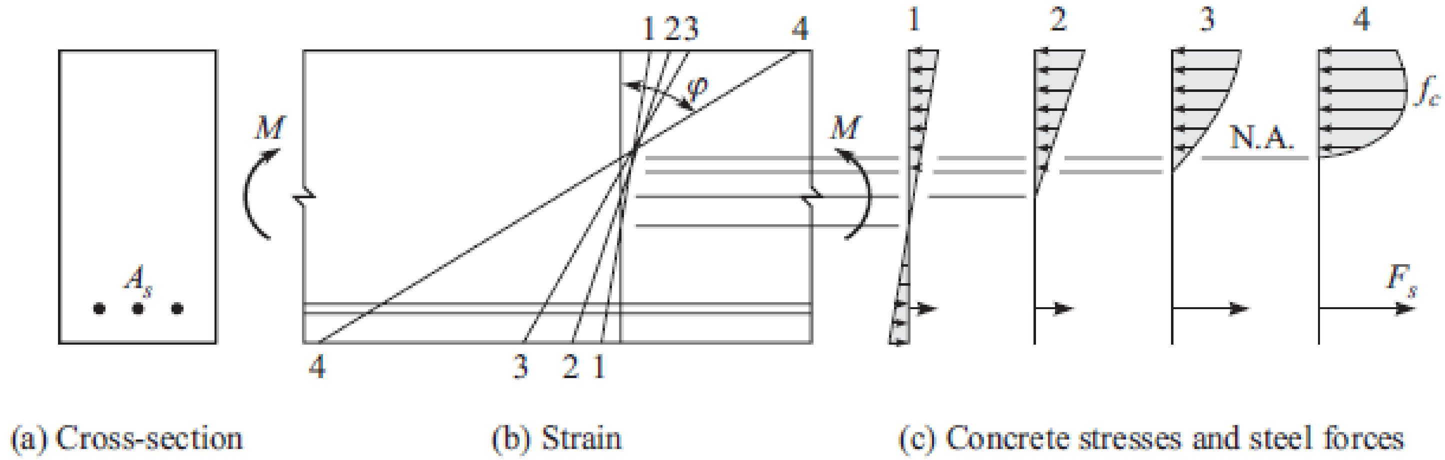


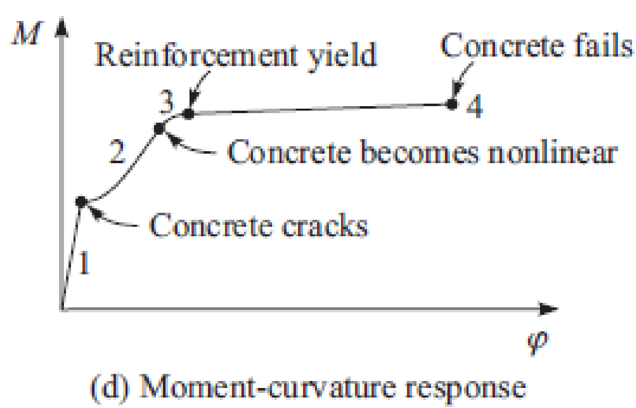
Stress-strain behavior of reinforced concrete in bending (Robberts and Marshall 2009)

FLEXURE ASSUMPTIONS FOR DESIGN

- Plane sections before bending remain plane after bending (Euler-Bernoulli's principle) – there is a linear strain distribution in the section, any deformation due to shear is negligible
- The stress-strain curve for the steel is known
- The tensile strength of the concrete may be neglected
- The stress-strain curve for concrete is known, defining the magnitude and distribution of the compressive stress

FLEXURE – STRESS AND STRAIN





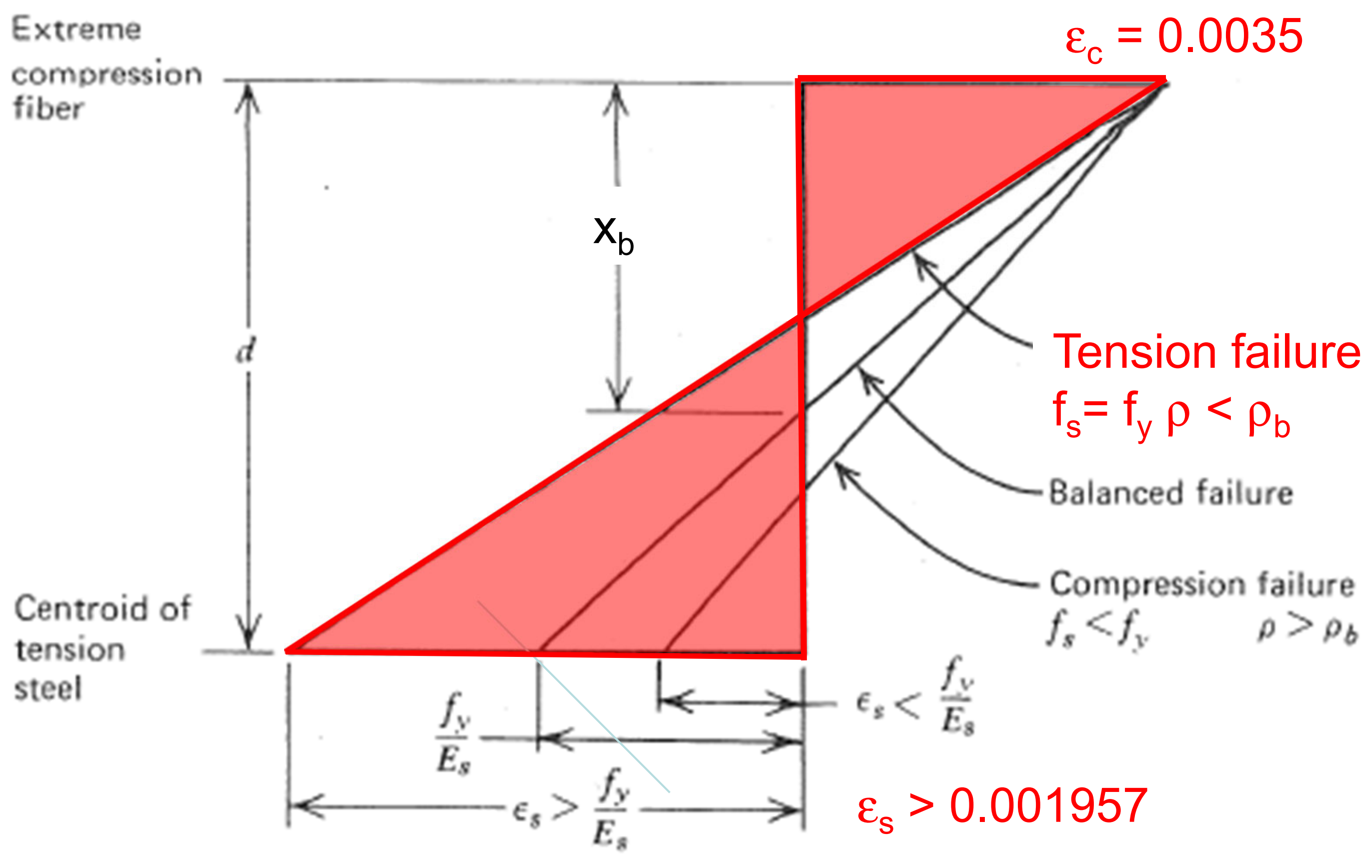
1 = Concrete is uncracked (linear elastic)
2 = Concrete is cracked (approximately linear elastic)
3 = Concrete stresses are nonlinear (plastic)
4 = Concrete fails in compression

Plane sections remain plane: there is a linear progression of strain through the section

Distribution of strains and stresses for flexure (Robberts and Marshall 2009)

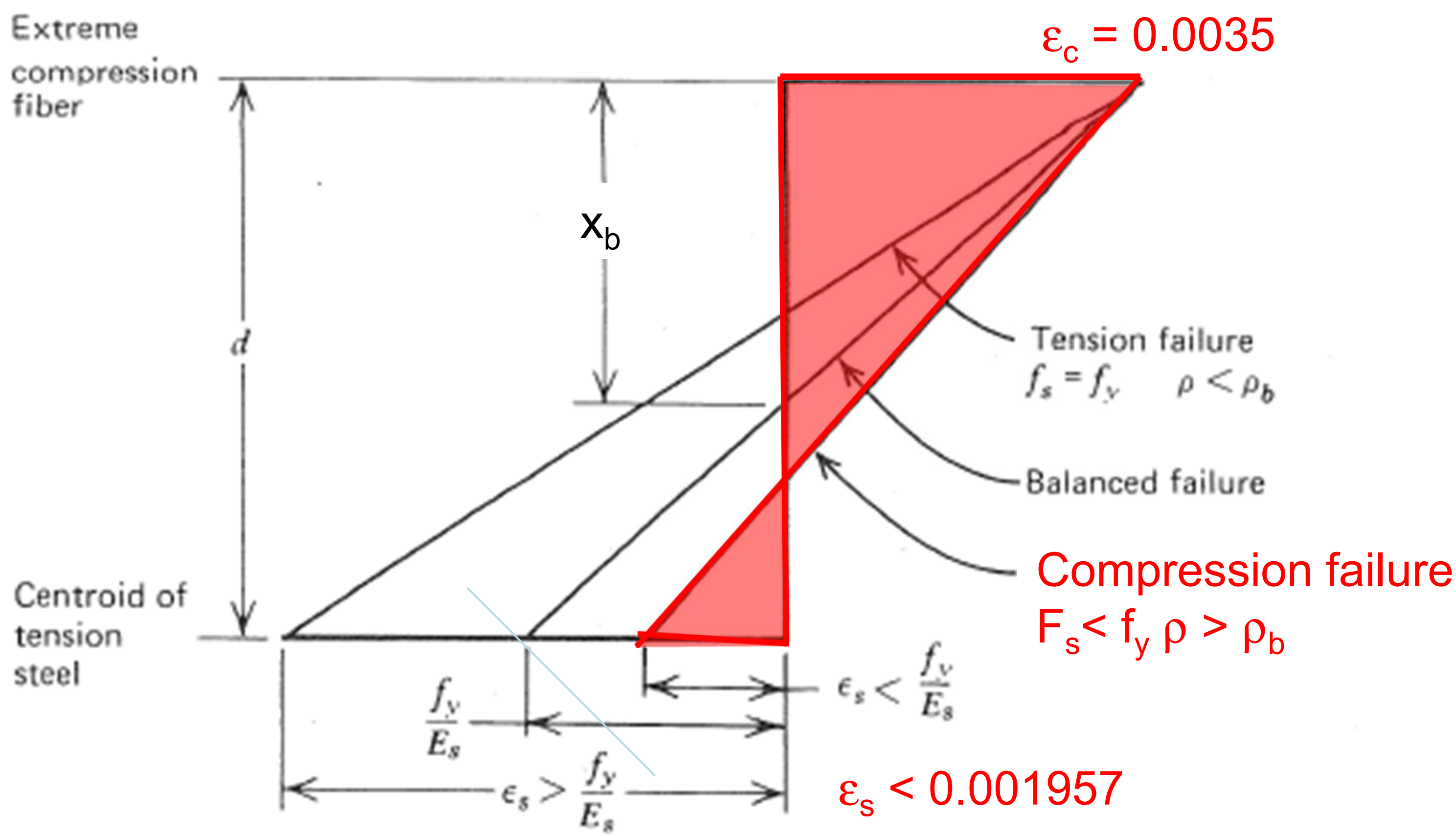
Curvature is a measure of how sharply a curve bends = the rate of change of the slope of a beam under load ($\phi = M / EI$)

TENSION FAILURE - UNDER REINFORCED



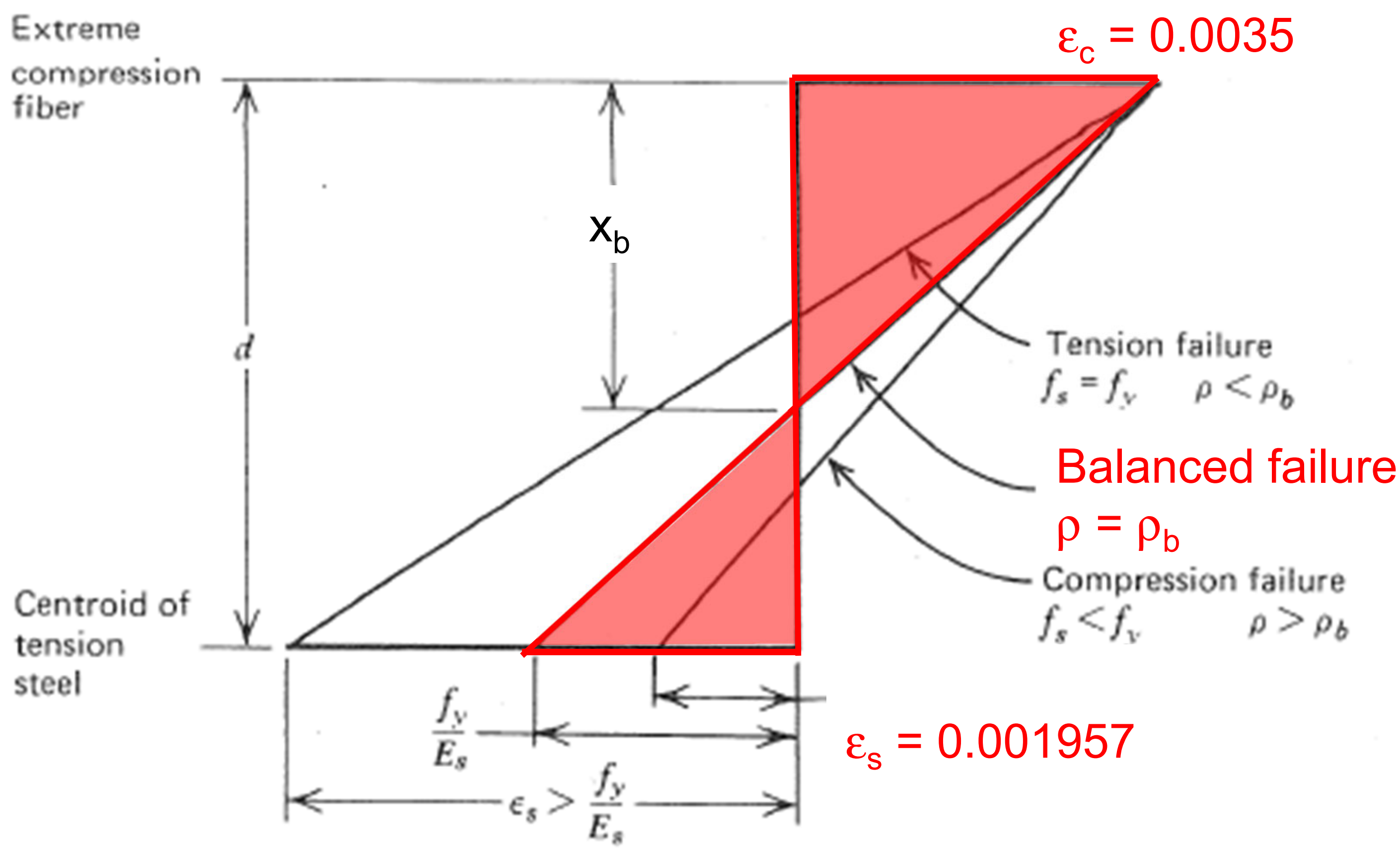
Strain profiles at the flexural strength of a section (Park and Paulay (1975))

COMPRESSION FAILURE - OVER REINFORCED



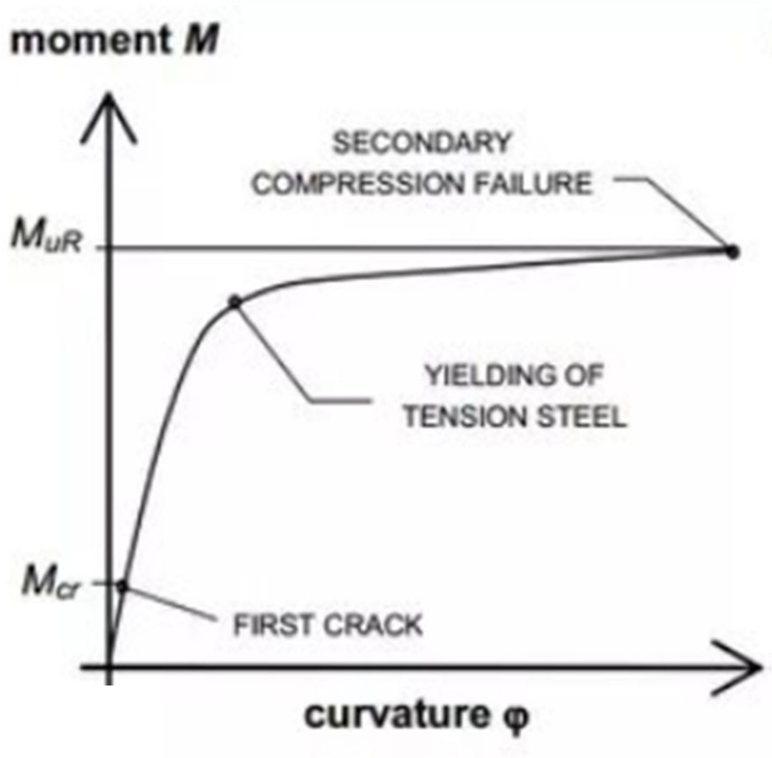
Strain profiles at the flexural strength of a section (Park and Paulay (1975))

BALANCED FAILURE

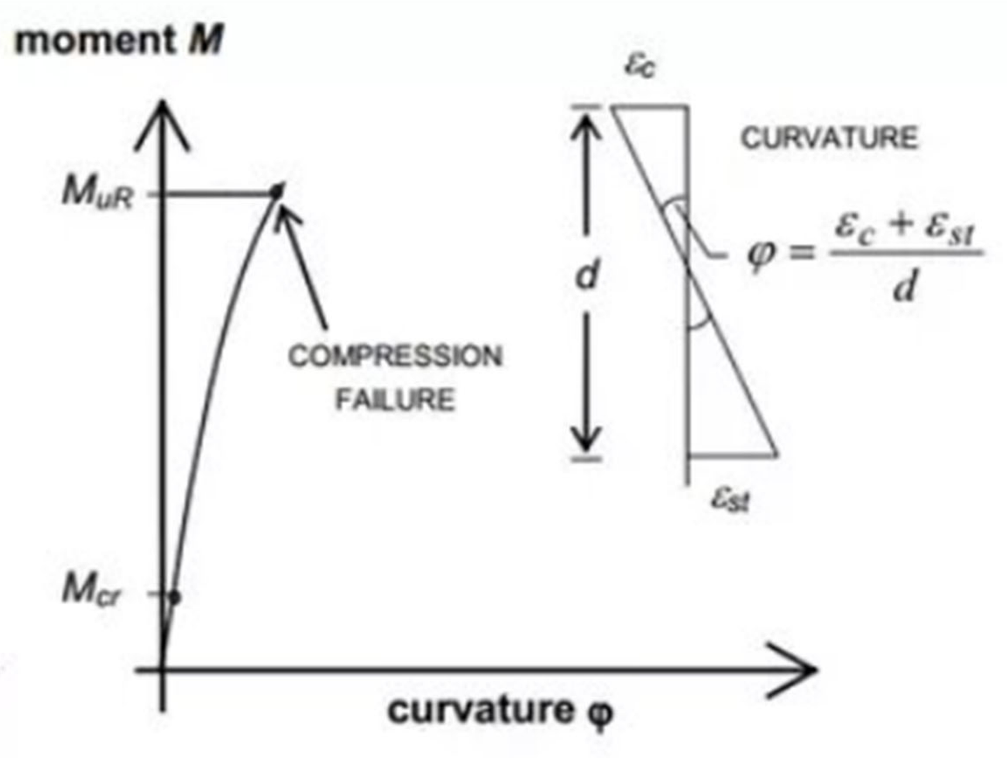


Strain profiles at the flexural strength of a section (Park and Paulay (1975))

UNDER VS OVER REINFORCED



Under-reinforced beam



Over-reinforced beam

Moment-curvature graphs for under vs over reinforced beams

DUCTILE FAILURE

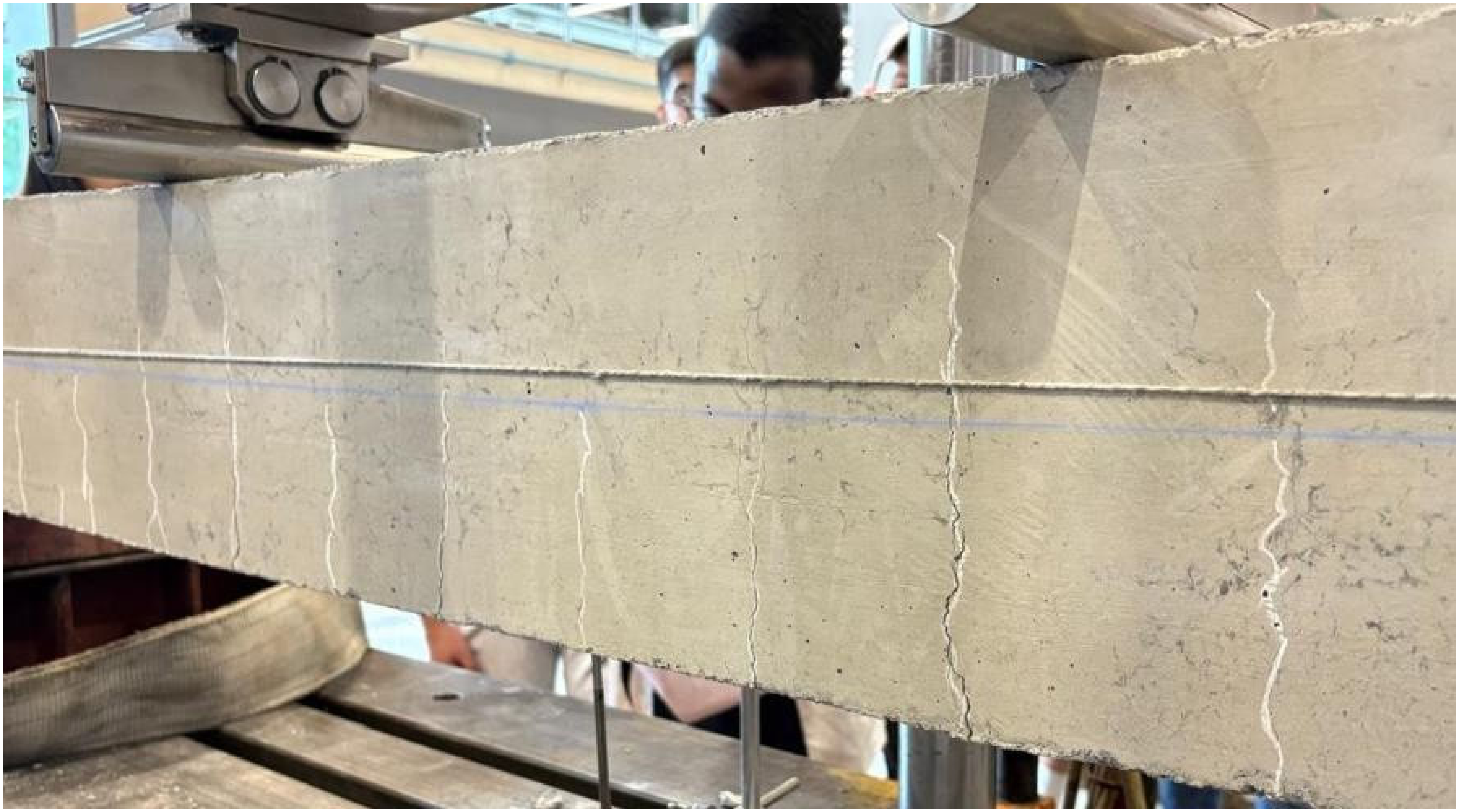


Photo: A Kleynhans

DUCTILE FAILURE

Photo: A Kleynhans

BRITTLE FAILURE

Photo: A Kleynhans

BRITTLE FAILURE

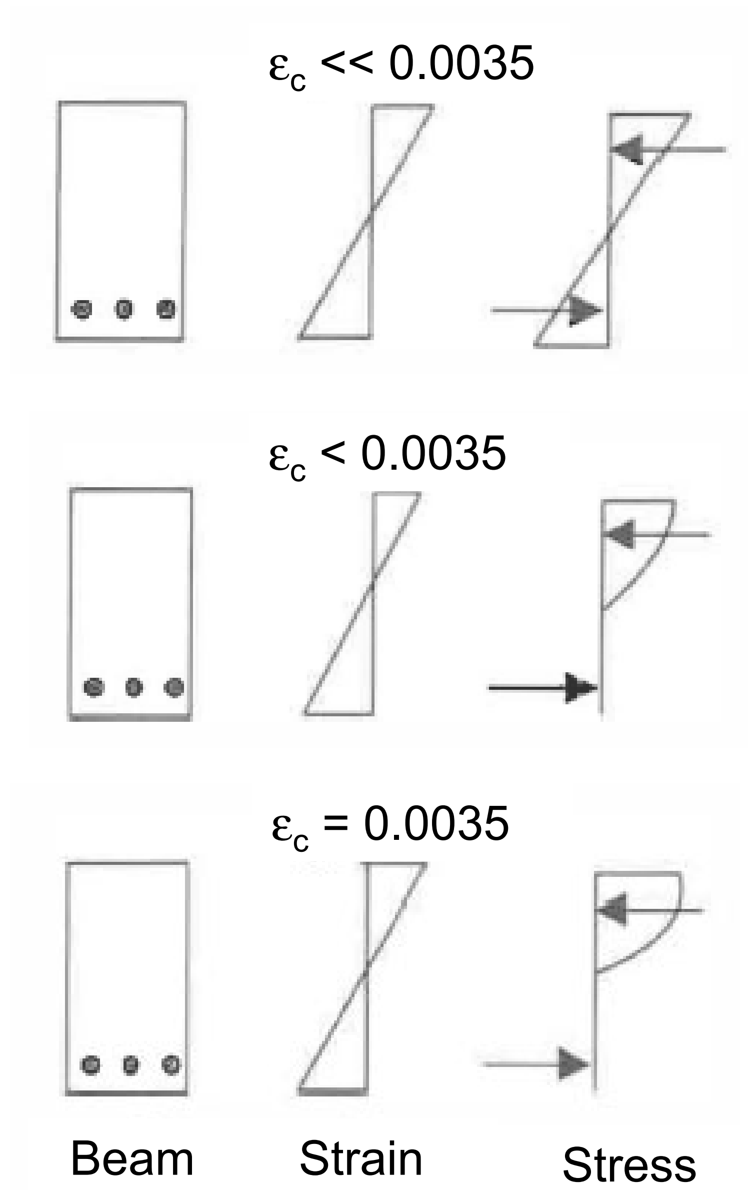
Photo: A Kleynhans

FLEXURE

$M < M_{CR}$

$M_{CR} < M < M_U$

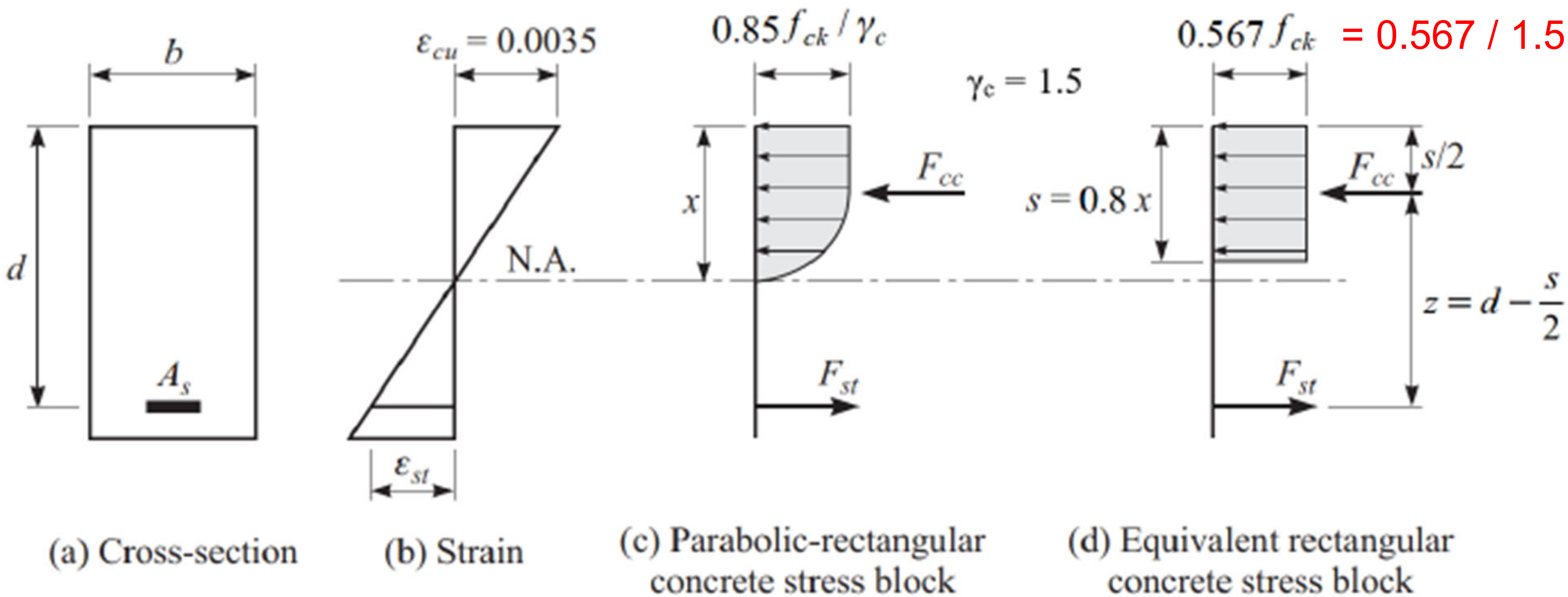
$M = M_U$



PARTIAL FACTORS OF SAFETY γ_M

Limit state	**Concrete**	**Steel**
ULS: Flexure, axial	1.5	1.15
ULS: Shear	1.4	1.15
ULS: Bond	1.4	NA
SLS	1.0	1.0

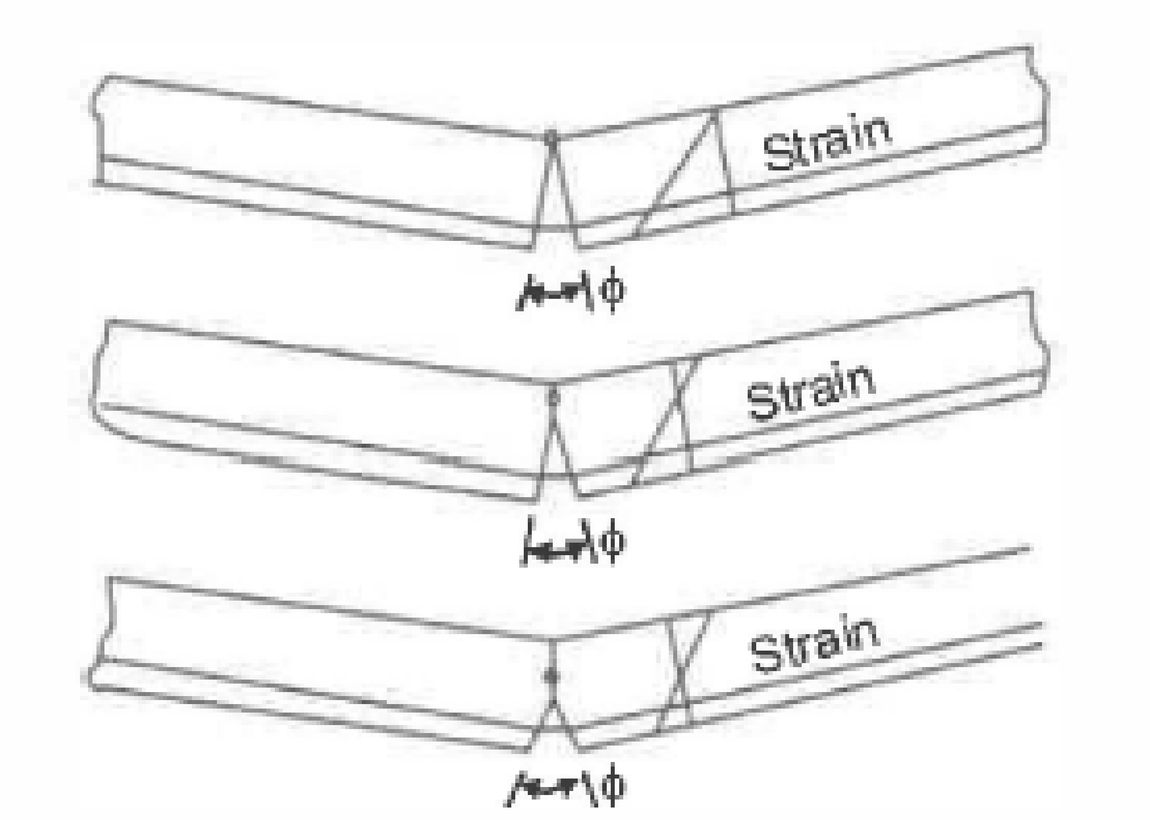
FLEXURE - EQUIVALENT STRESS BLOCK



Equivalent stress block for singly reinforced rectangular sections

Limit: $x \leq 0.45\,d$

FLEXURAL DUCTILITY

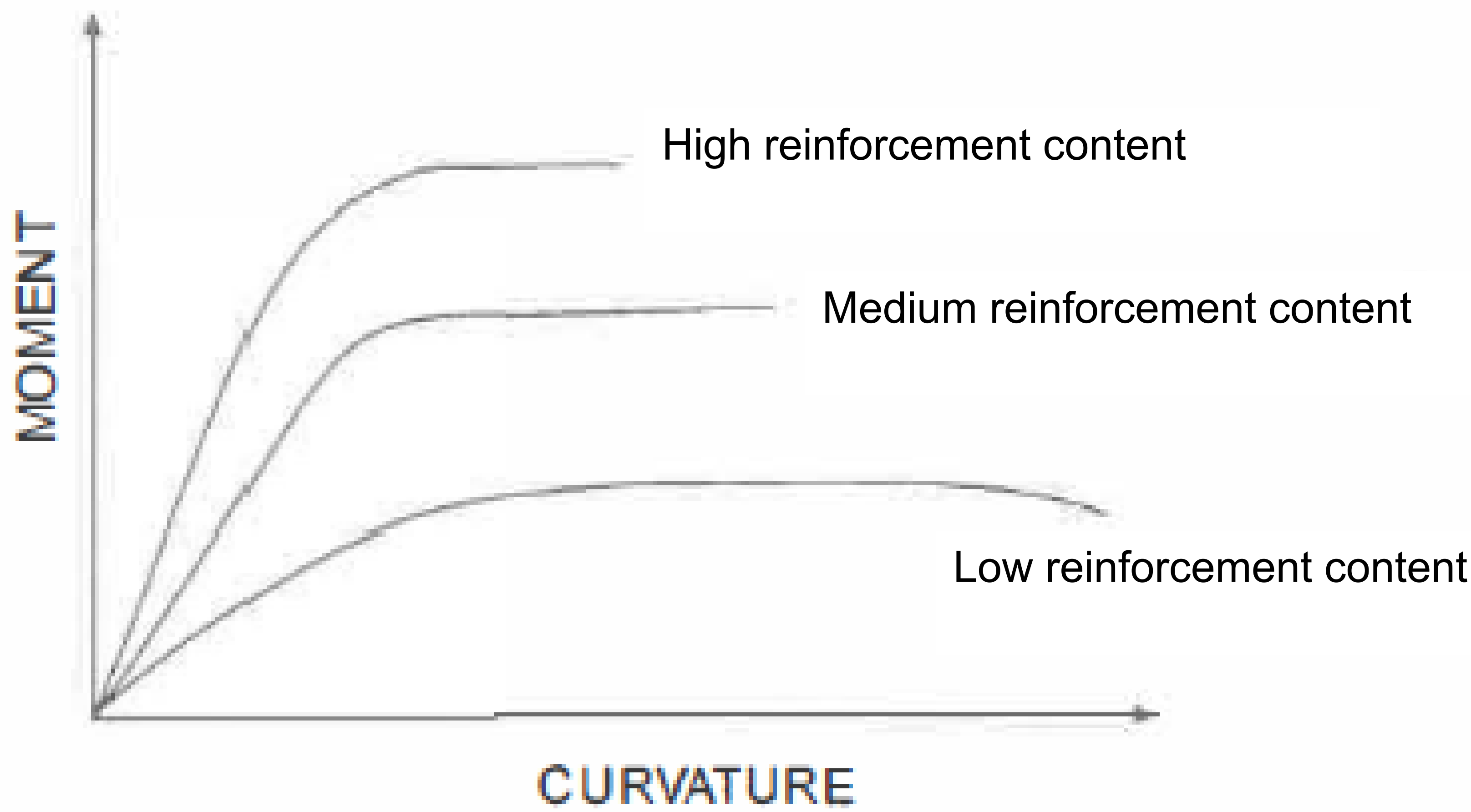


Low reinforcement content

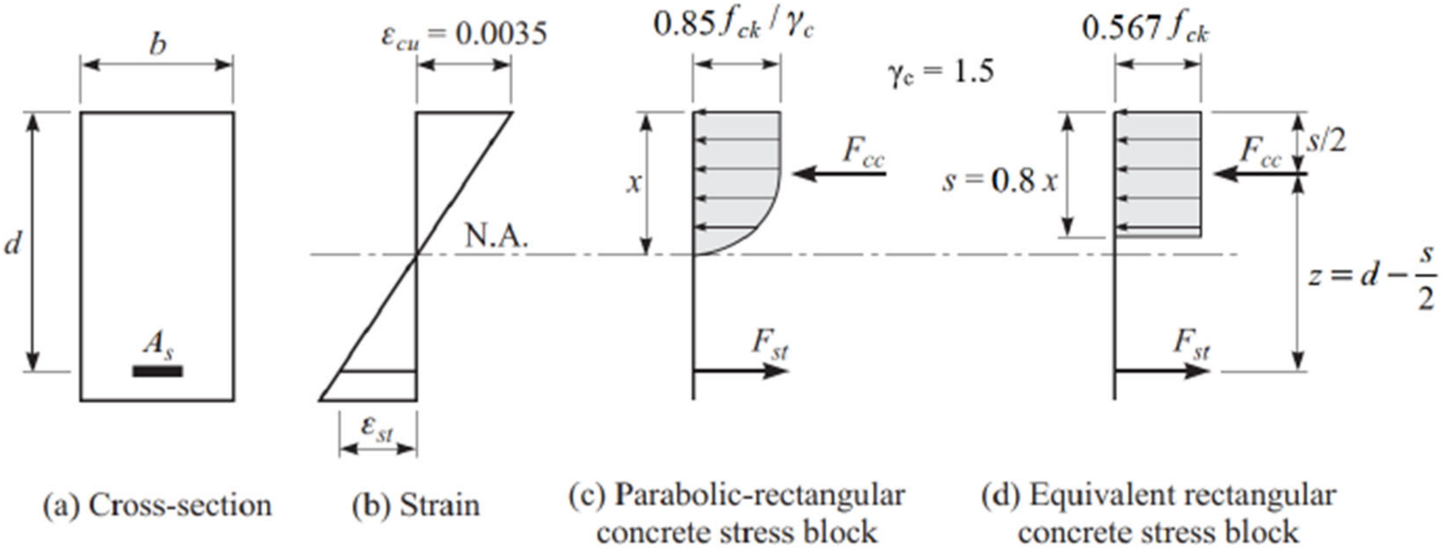
Medium reinforcement content

High reinforcement content

FLEXURAL DUCTILITY



DESIGN OF SINGLY REINFORCED SECTIONS



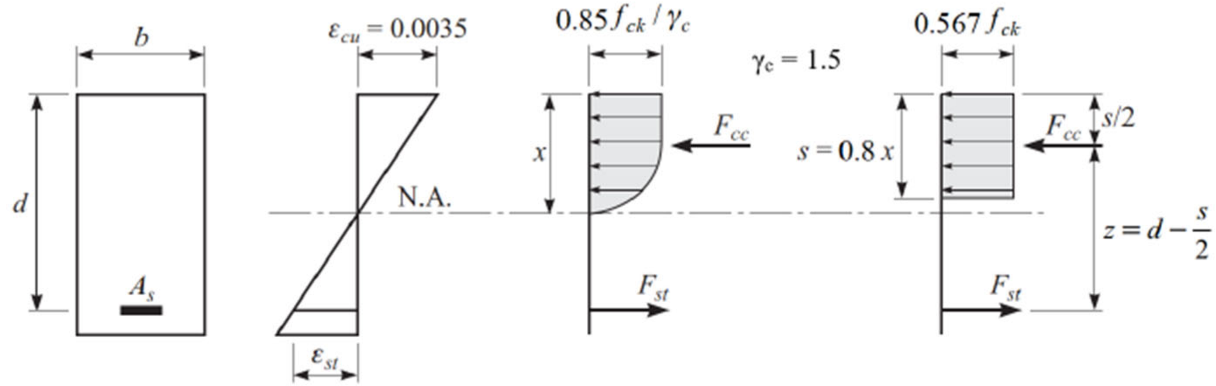
In order to achieve the desired level of flexural ductility, i.e rotation capacity, the depth of the neutral axis is limited to x = 0.45d. This limit is often referred to as a balanced section and corresponds roughly to 1.2 % of reinforcing.

1) Calculate the limiting moment that can be resisted without compression reinforcing:

$M_{max} = 0.167bd^2 f_{ck}$ or $M_{max} = 0.132bd^2 f_{cu}$

.

DESIGN OF SINGLY REINFORCED SECTIONS



(a) Cross-section (b) Strain (c) Parabolic-rectangular concrete stress block (d) Equivalent rectangular concrete stress block

2) If the applied moment is less than this value then calculate K:

$$K = \frac{M}{bd^2 f_{ck}} < K' = 0.167$$ (neutral axis depth factor)

And calculate the lever arm

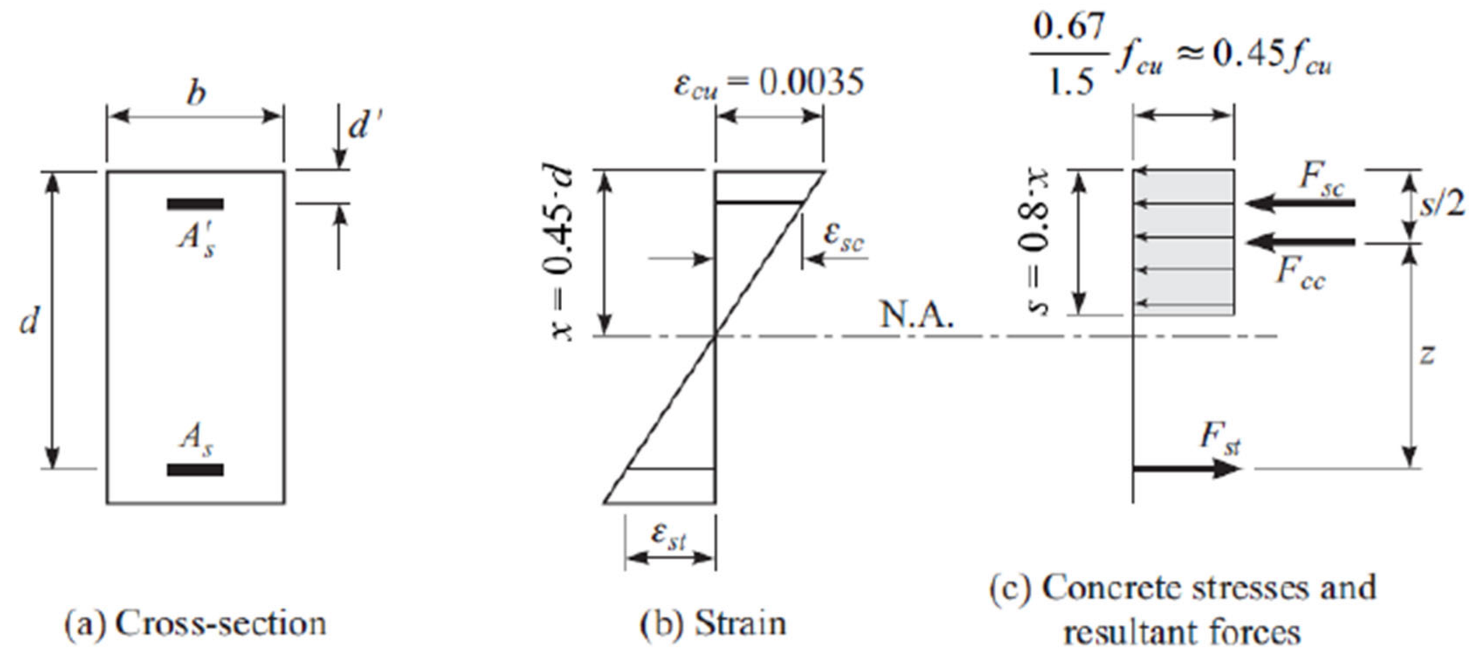
$$z = d\left(0.5 + \sqrt{0.25 - {}^{K}/_{1.134}}\right) \le 0.95d$$

Then the required reinforcement is:

$$A_s = \frac{M}{0.87 f_y z}$$

If the applied moment is more than this value then design for compression reinforcement.

DOUBLY REINFORCED SECTIONS



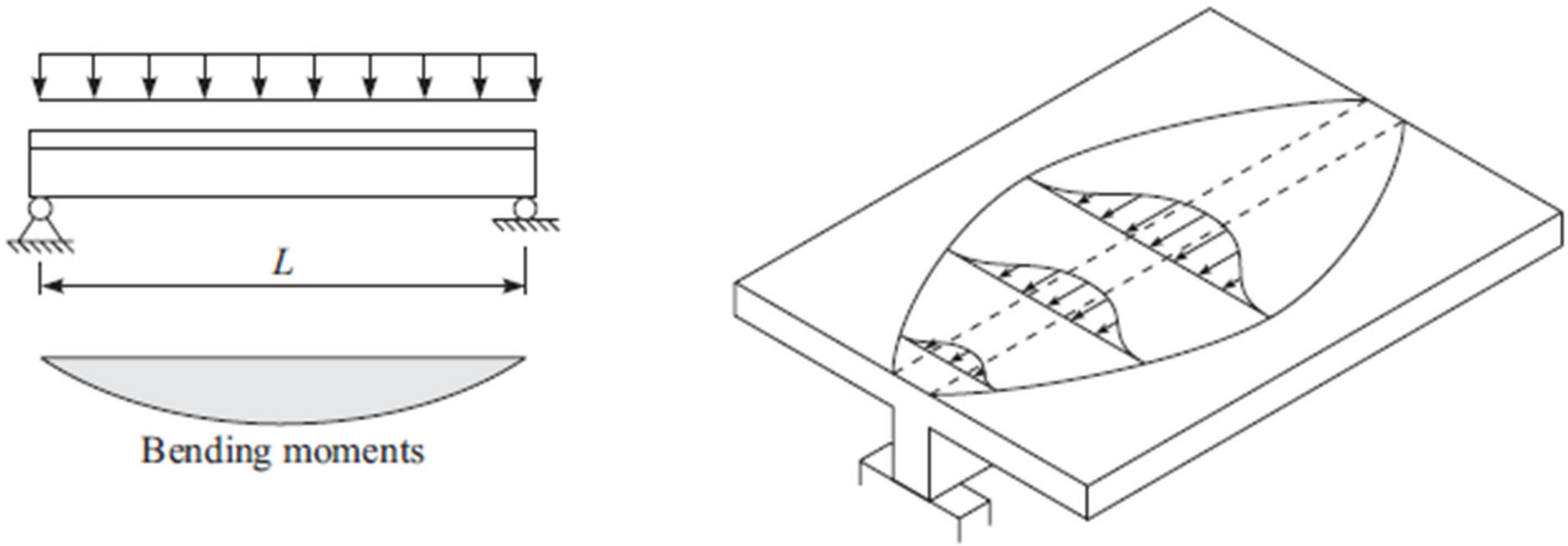
Horizontal equilibrium:

$F_{st} = F_{sc} + F_{cc}$

The total moment of resistance may be expressed as:

$M_r = M_c + M_{sc}$

T SECTIONS

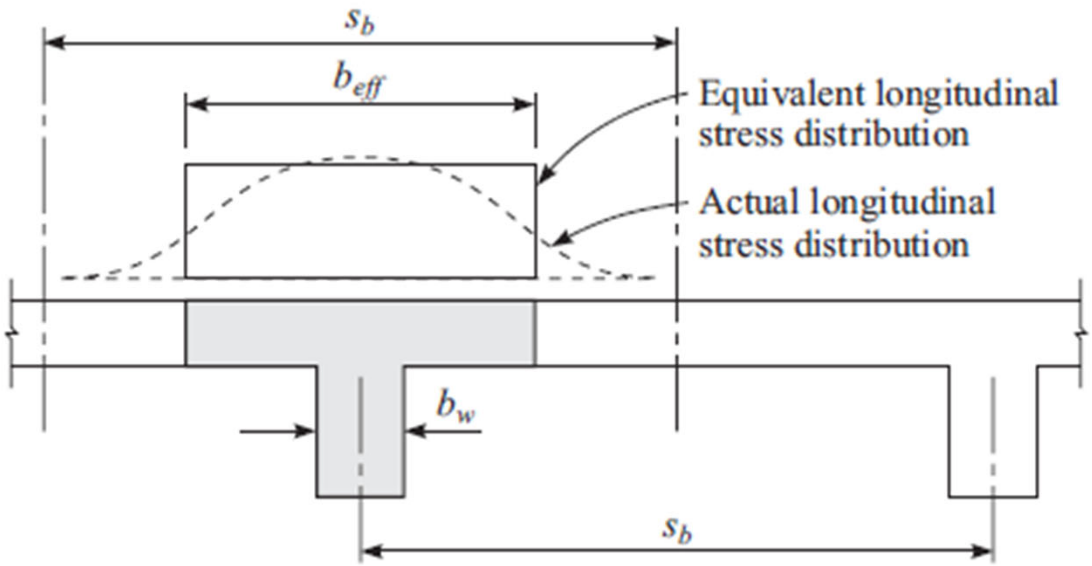


(a) Simply supported beam

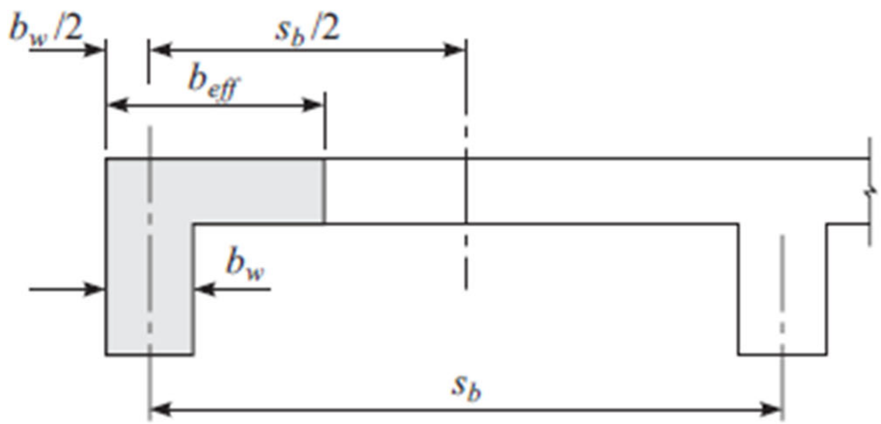
(b) Distribution of longitudinal compressive stresses

Effective flange width of a simply supported T beam (Robberts and Marshall 2009)

T SECTIONS



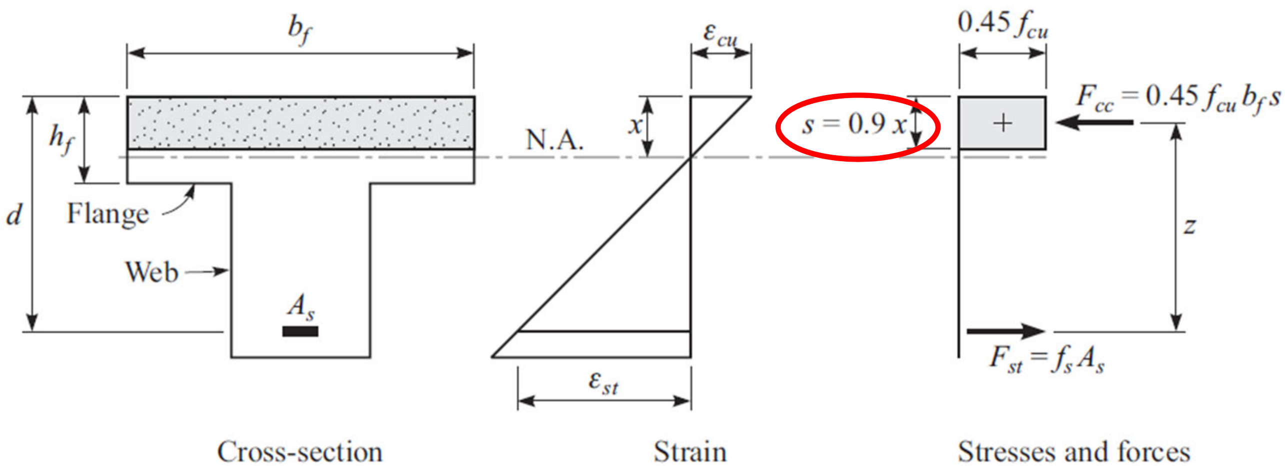
(a) Effective width of a T-section



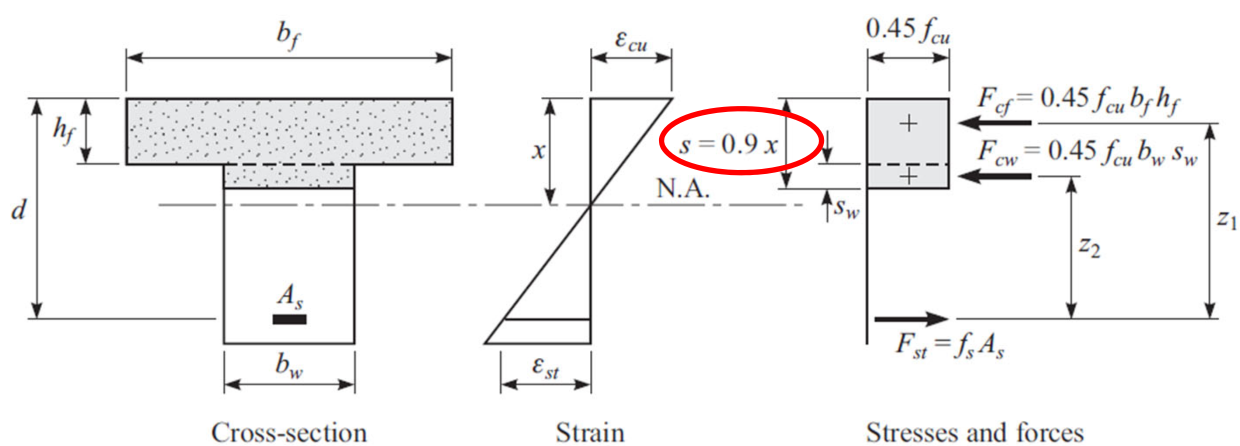
(b) Effective width of an L-section

Effective flange width of a T and L beam (Robberts and Marshall 2009)

T SECTIONS



T-section with stress block in flange (Robberts and Marshall 2009)



T-section with stress block in web (Robberts and Marshall 2009)

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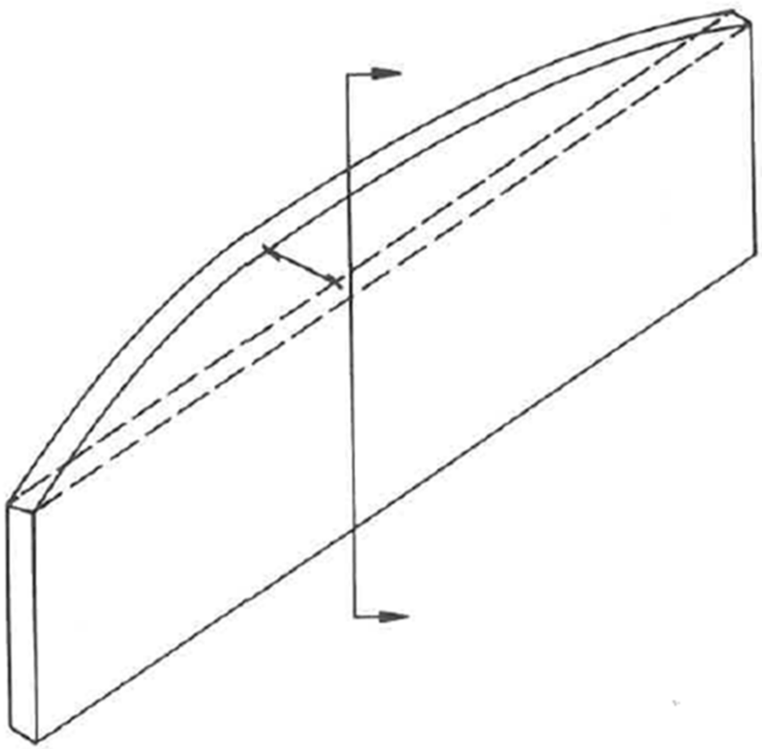
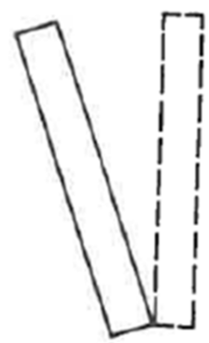
CONCRETE BEAM BUCKLING

The limit typically used where concrete beams are susceptible to buckling:

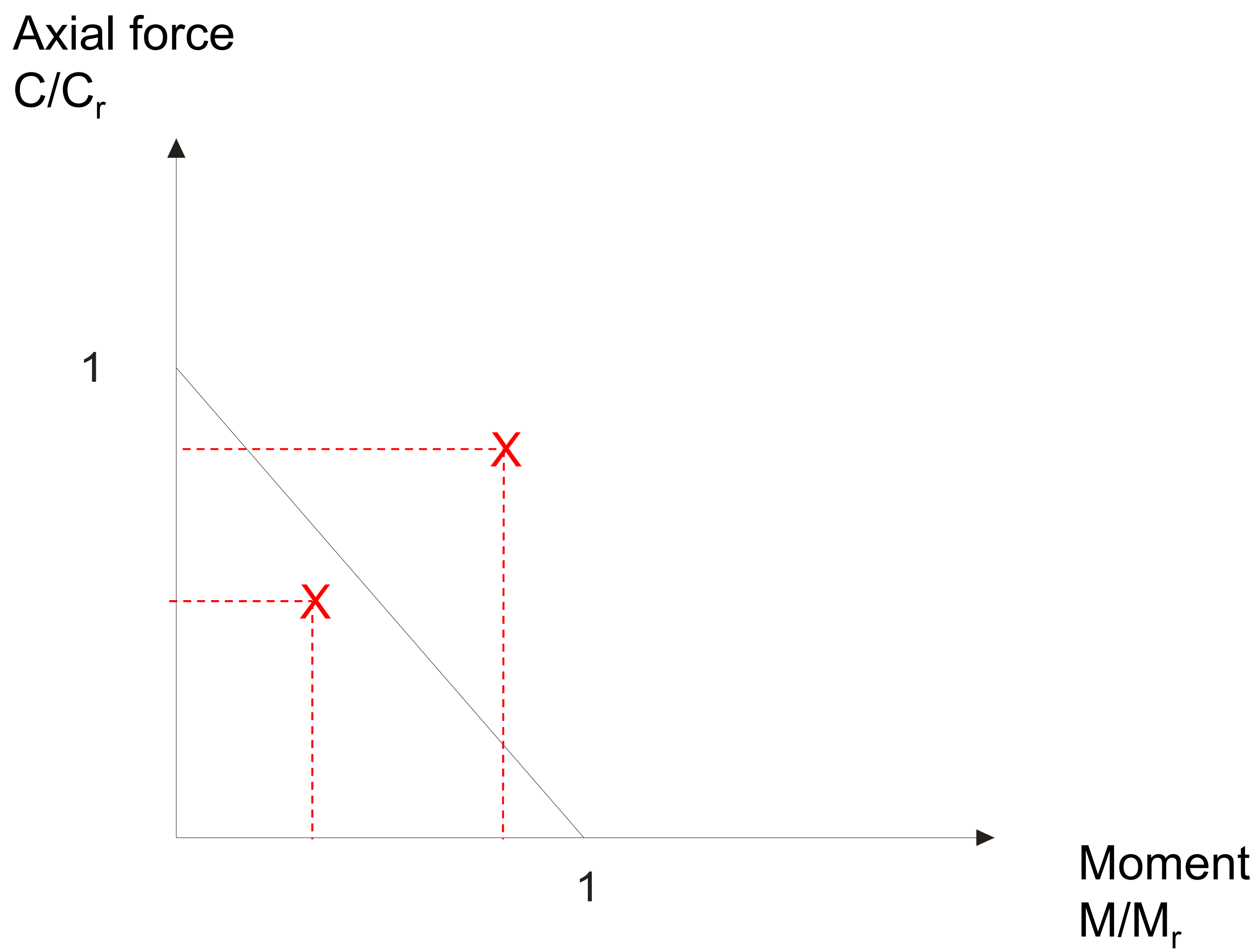
- L / b > 60 for simply supported or continuous beams
- L / b > 25 for cantilevers
- L = distance between lateral restraints



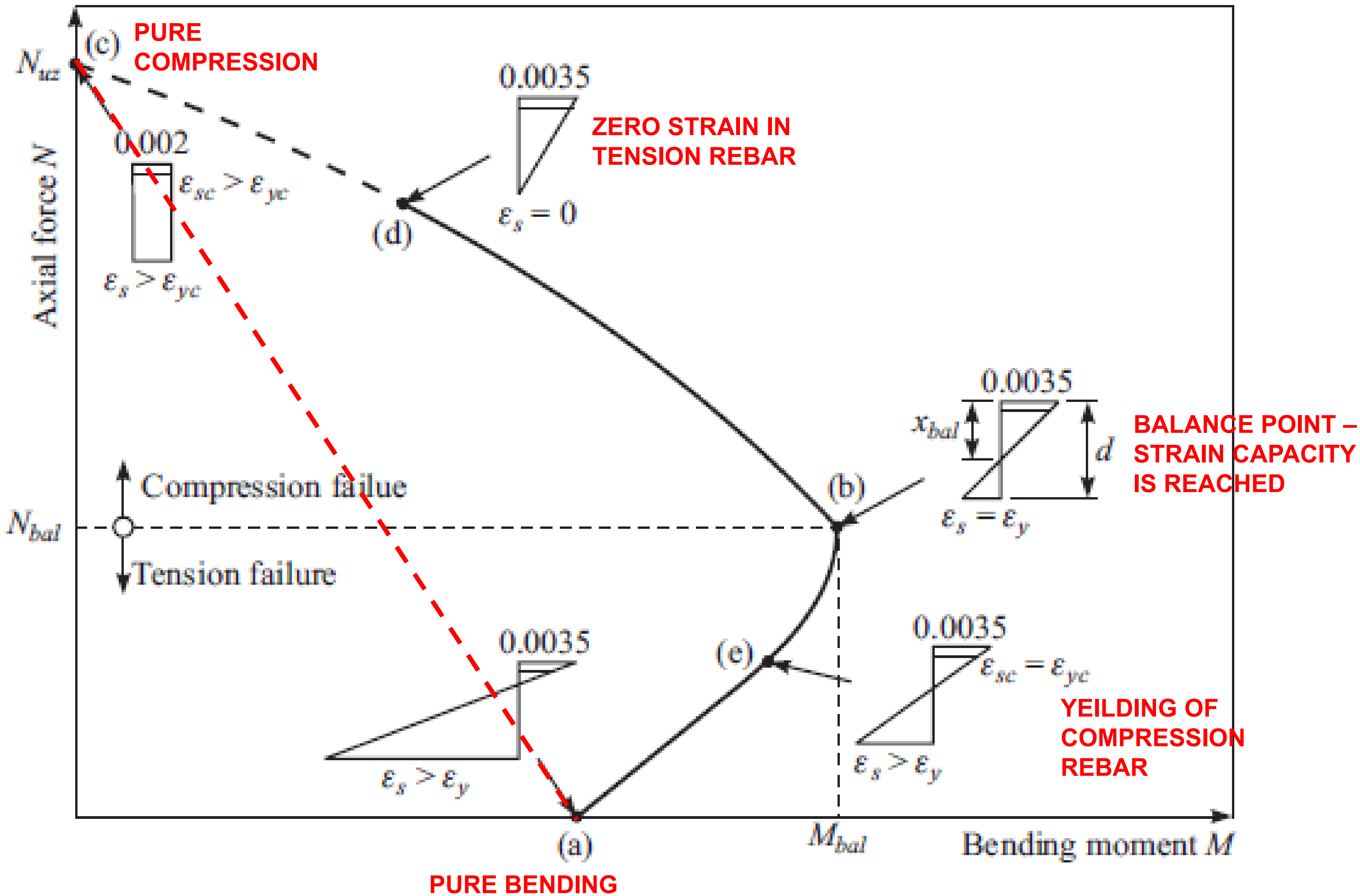


Midspan deflection

FLEXURE AND AXIAL FORCE



FLEXURE AND AXIAL FORCE



FLEXURE AND AXIAL FORCE

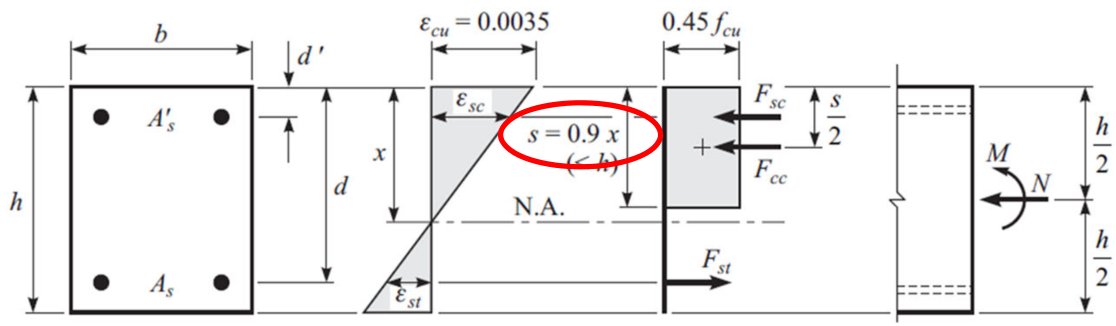
Horizontal equilibrium:

$$N = F_{cc} + F_{sc} + F_{st}$$
$$= 0.45 f_{cu} b s + f_{sc} A'_s + f_s A_s$$

Moment equilibrium:

(take moments about N)

$$M = F_{cc}\left(\frac{h}{2} - \frac{s}{2}\right) + F_{sc}\left(\frac{h}{2} - d'\right) - F_{st}\left(d - \frac{h}{2}\right)$$
$$= 0.45 f_{cu} b s\left(\frac{h}{2} - \frac{s}{2}\right) + f_{sc} A'_s\left(\frac{h}{2} - d'\right) - f_s A_s\left(d - \frac{h}{2}\right)$$



$$\frac{N}{bh} = 0.45 f_{cu}\left(\frac{0.9x}{h}\right) + \frac{f_{sc}}{2}\left(\frac{A_{sc}}{bh}\right) + \frac{f_s}{2}\left(\frac{A_{sc}}{bh}\right)$$

$$\frac{M}{bh^2} = 0.225 f_{cu}\left(\frac{0.9x}{h}\right)\left(1 - \frac{0.9x}{h}\right) + \frac{f_{sc}}{2}\left(\frac{A_{sc}}{bh}\right)\left(\frac{1}{2} - \frac{d'}{h}\right) - \frac{f_s}{2}\left(\frac{A_{sc}}{bh}\right)\left(\frac{d}{h} - \frac{1}{2}\right)$$

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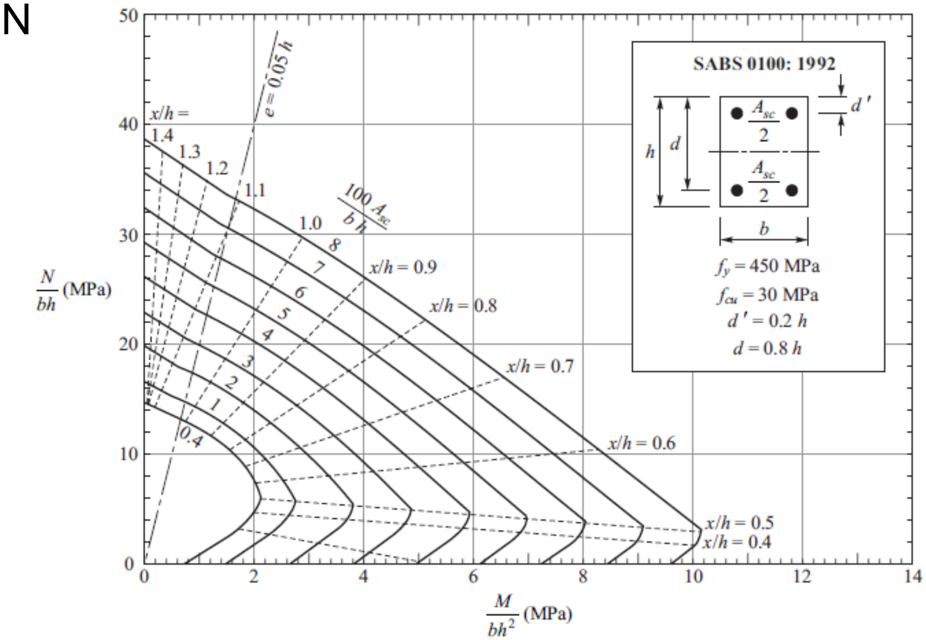
INTERACTION DIAGRAMS

These equations can be used to produce a M-N interaction diagram design chart:

- Choose values for f_{cu}, f_y, d/h and d'/h
- Choose a value for $A_{sc}/(bh)$
- Choose a value for x/h
- Calculate stains (ε_s) and stresses f_s in the rebar
- Calculate N/(bh) and $M/(bh^2)$
- Repeat these steps for different values of x/h – this will produce one curve for a given $A_{sc}/(bh)$ on the interaction diagram
- Repeat steps for different values of $A_{sc}/(bh)$ to generate complete M-N diagram



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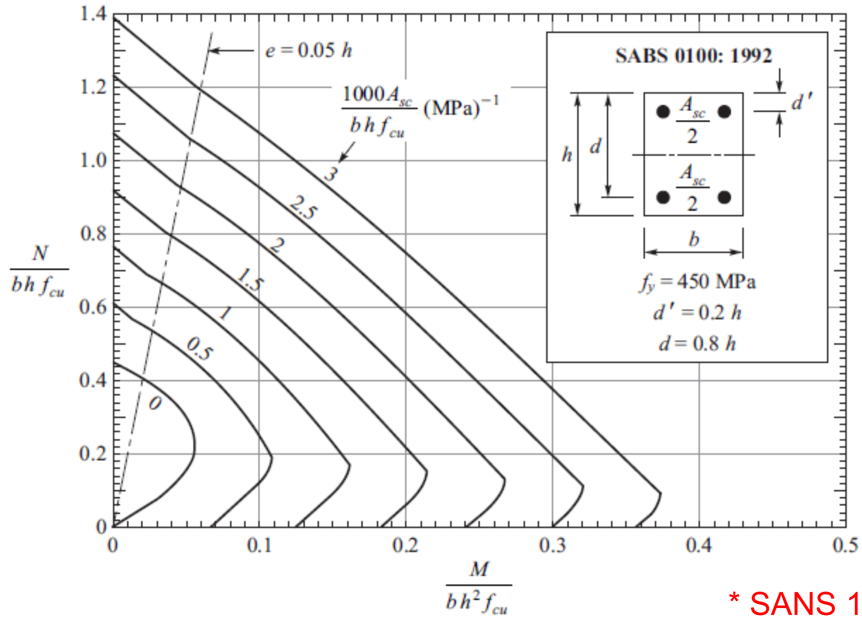
INTERACTION DIAGRAMS

To reduce the number of charts required to cover all possible values of f_{cu}:

$$\frac{N}{bh\,f_{cu}} = 0.45\left(\frac{0.9x}{h}\right) + \frac{f_{sc}}{2}\left(\frac{A_{sc}}{bh\,f_{cu}}\right) + \frac{f_s}{2}\left(\frac{A_{sc}}{bh\,f_{cu}}\right)$$

$$\frac{M}{bh^2 f_{cu}} = 0.225\left(\frac{0.9x}{h}\right)\left(1 - \frac{0.9x}{h}\right) + \frac{f_{sc}}{2}\left(\frac{A_{sc}}{bh\,f_{cu}}\right)\left(\frac{1}{2} - \frac{d'}{h}\right) - \frac{f_s}{2}\left(\frac{A_{sc}}{bh\,f_{cu}}\right)\left(\frac{d}{h} - \frac{1}{2}\right)$$

f_{cu} is now included with the area of the reinforcement and section dimensions as the independent variable.



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INTERACTION DIAGRAMS

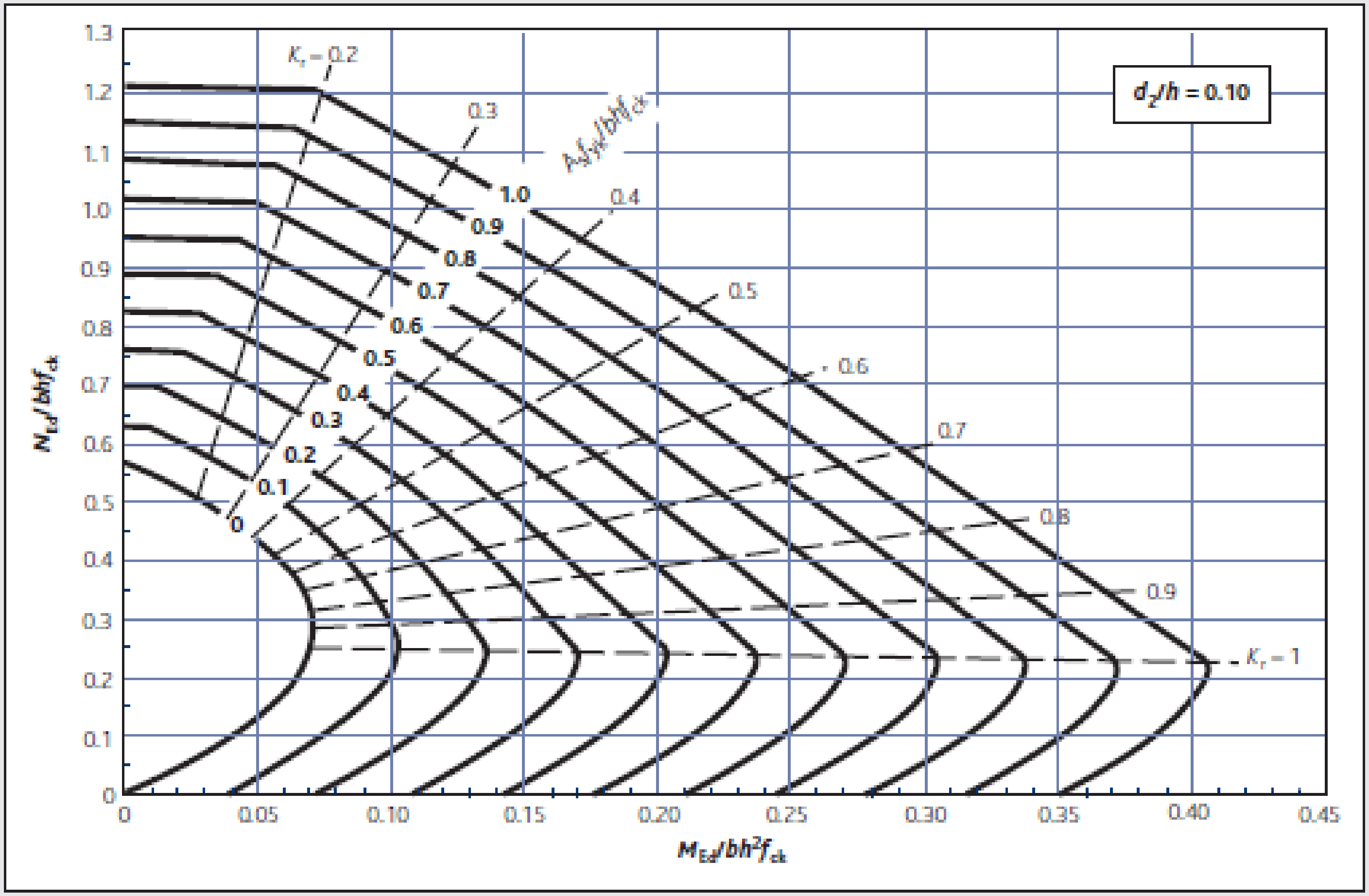
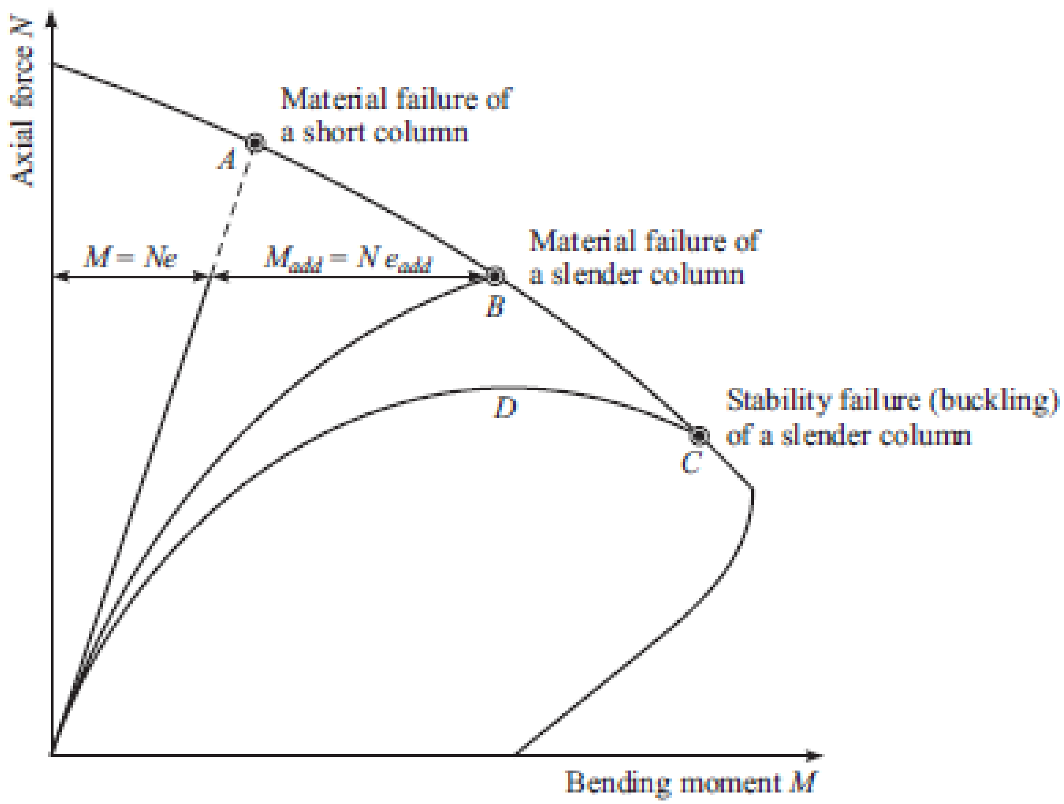
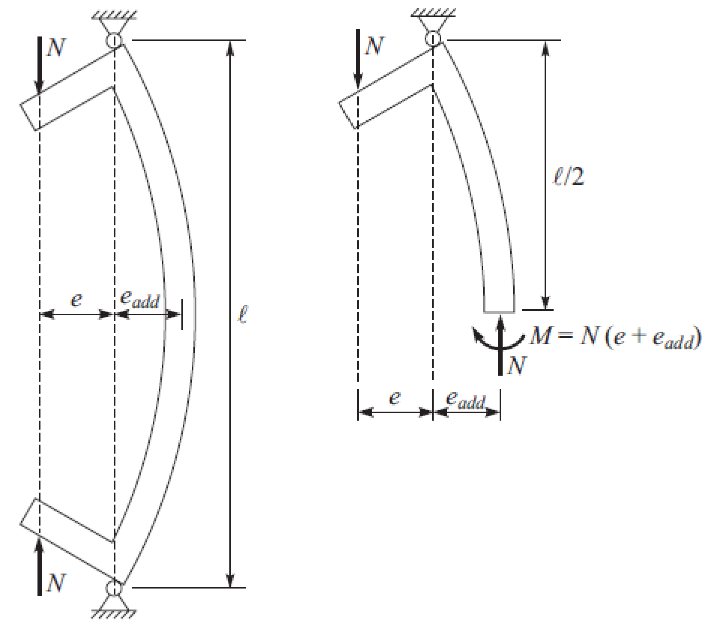


Figure 15.5b)
Rectangular columns $d_2/h = 0.10$

SLENDER COLUMNS





Additional moments caused by slenderness

Influence of slenderness on the failure mode (Robberts and Marshall 2009)

CONCLUSIONS

What are the important principals that we need to consider for reinforced concrete flexural design?

- Load effects – M, V, T, N
- Stress and strain – normal / shear
- Ductility
- Under-reinforced versus over-reinforced concrete beams
- Shear lag
- Equilibrium of forces, compatibility of strain and known stress-strain relationships are used to set up the design equations

REFERENCES

Reinforced Concrete Structures
Park and Paulay (1975)

Analysis and Design of Reinforced Concrete Structures
Robberts and Marshall (2009)

Reinforced concrete design to Eurocode 2
Mosley, Bungey and Hulse (2012)

SANS 51992-1-1
The new standard for the design of concrete structures in South Africa, adopted from the Eurocode 2 standard.