

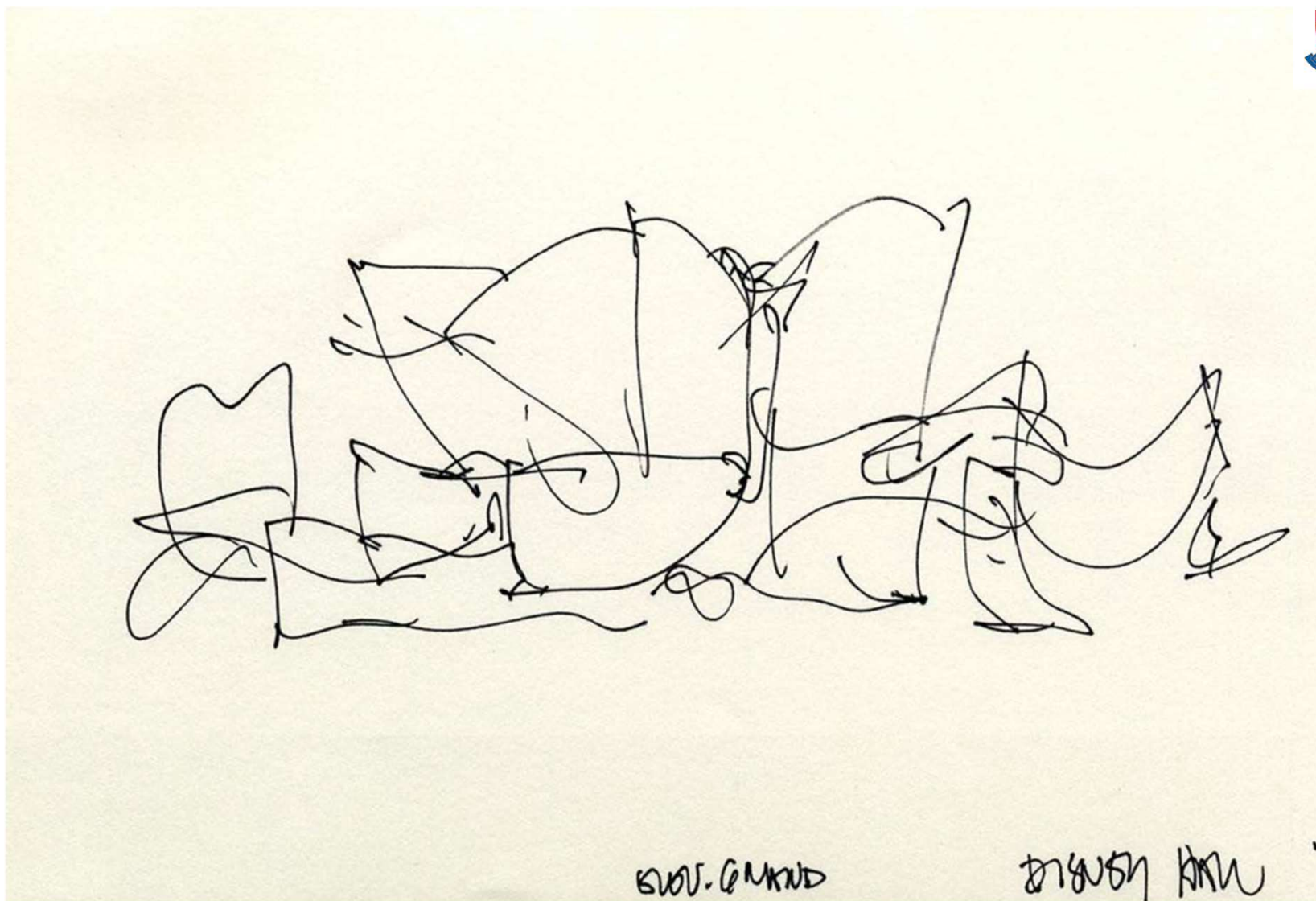
An Introduction to Graphic Statics and its Relation to Optimal Structural Form



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Why design for optimal form?



Walt Disney Concert Hall, Los Angeles, 2003 | Frank Gehry



Walt Disney Concert Hall, Los Angeles, 2003 | Frank Gehry



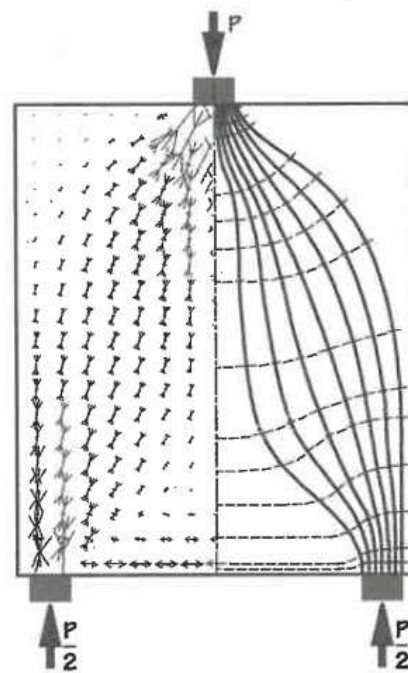
Walt Disney Concert Hall, Los Angeles, 2003 | Frank Gehry

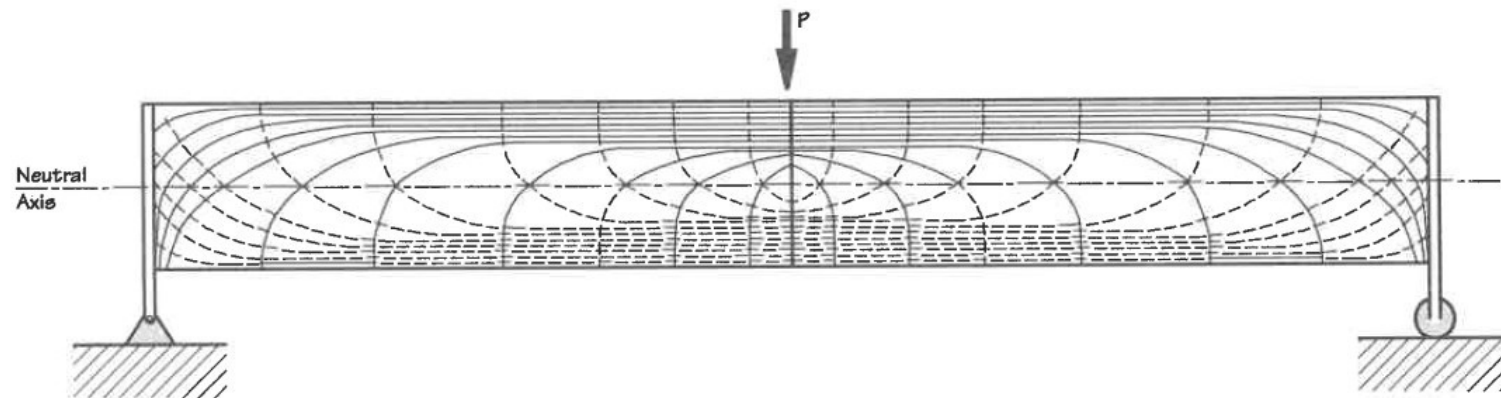


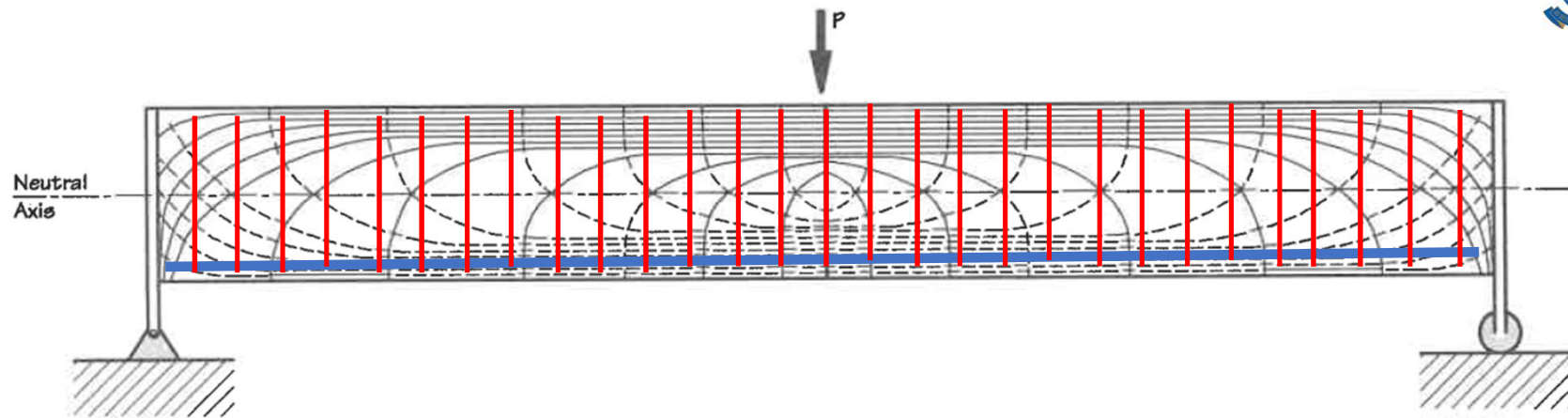


How forces (want to) flow

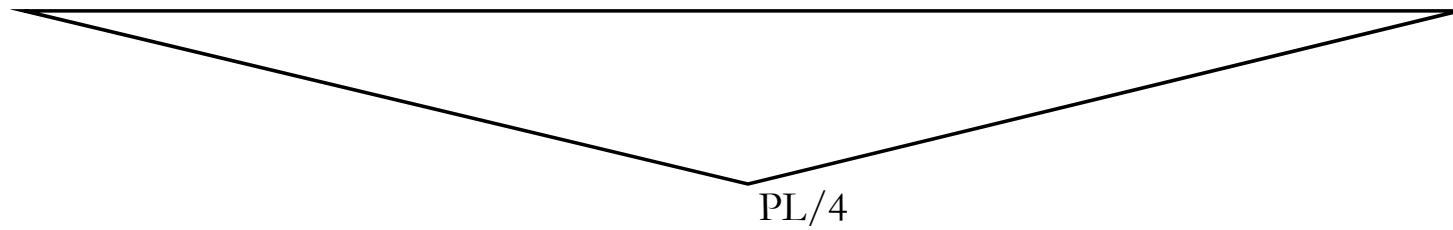




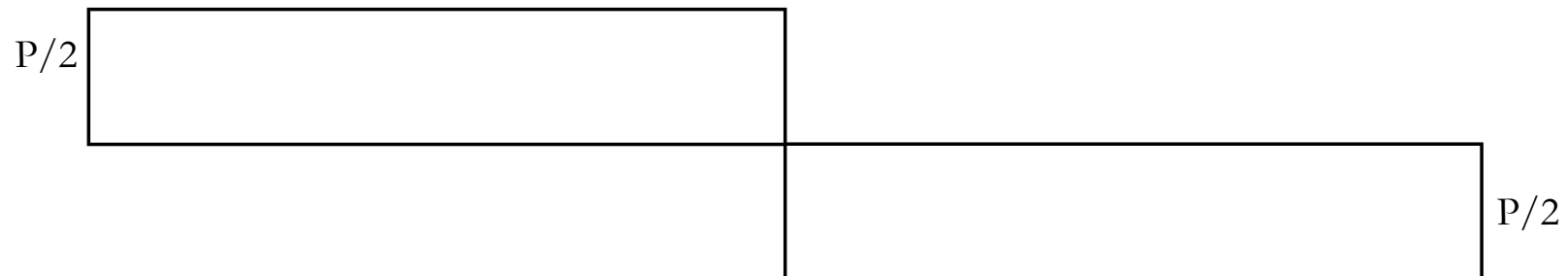




Bending moment diagram



Shear force diagram



In the above formula, by the term *statical moment* is meant the product of the area mentioned by the distance between its center of gravity and the neutral axis. For example, the longitudinal shearing intensity at a point *c* in a rectangular beam, Fig. 2, may be expressed as follows:

$$v = \frac{VA'x}{Ib}$$

For rectangular beams and all beams of uniform width, the largest value of *v* for any given section will occur at the neutral axis since the statical moment *Q* has its maximum value for a point on this axis, and *b* is constant.

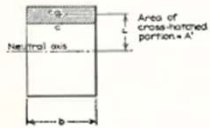


FIG. 2.

6. If a beam is of constant cross-section throughout, the maximum values of *f* and *v* will occur at the section where *M* and *V* respectively have maximum values.

7. In addition to the longitudinal or horizontal shear at any point there coexists a vertical shear and the intensity of this vertical shear is equal to the intensity of the horizontal shear.

8. The intensity of the shear at the top and bottom of a beam is zero and the intensity of shear (horizontal and vertical) along a vertical cross-section for a rectangular beam varies as the ordinates to a parabola, as shown graphically in Fig. 3. The maximum value occurs at the neutral axis and is $\frac{3}{2}$ the average intensity, or $\frac{3}{2} \cdot \frac{V}{bd}$.

9. At the neutral plane there exists a tension and compression at angles of 45 deg. to the horizontal, and the intensity of these forces is equal to that of the shear.

10. At the end of a simply supported beam where the shear is a maximum and the bending moment a minimum, the stresses lie practically at 45 deg. to the horizontal throughout the entire depth of beam.

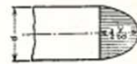


FIG. 3.

11. At the section of maximum moment, the shear is zero and the stresses are horizontal.

12. If *f* represents the intensity of horizontal fiber stress and *v* the intensity of vertical or horizontal shearing stress at any point in a beam, the intensity of the inclined stress will be given by the formula

$$t = \frac{1}{2}f \pm \sqrt{\frac{1}{4}f^2 + v^2}$$

and the direction of this stress by the formula

$$\tan 2K = \frac{2v}{f}$$

where *K* is the angle of the stress with the horizontal.

13. At any given point maximum compressive stress and maximum tensile stress make an angle of 90 deg. with each other.

14. The directions of the maximum stresses for a simply supported beam uniformly loaded are as given in Fig. 4. The general direction of the stresses in a beam with any given loading may be determined by means of the formulas for *t* and *K* given above.

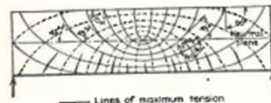
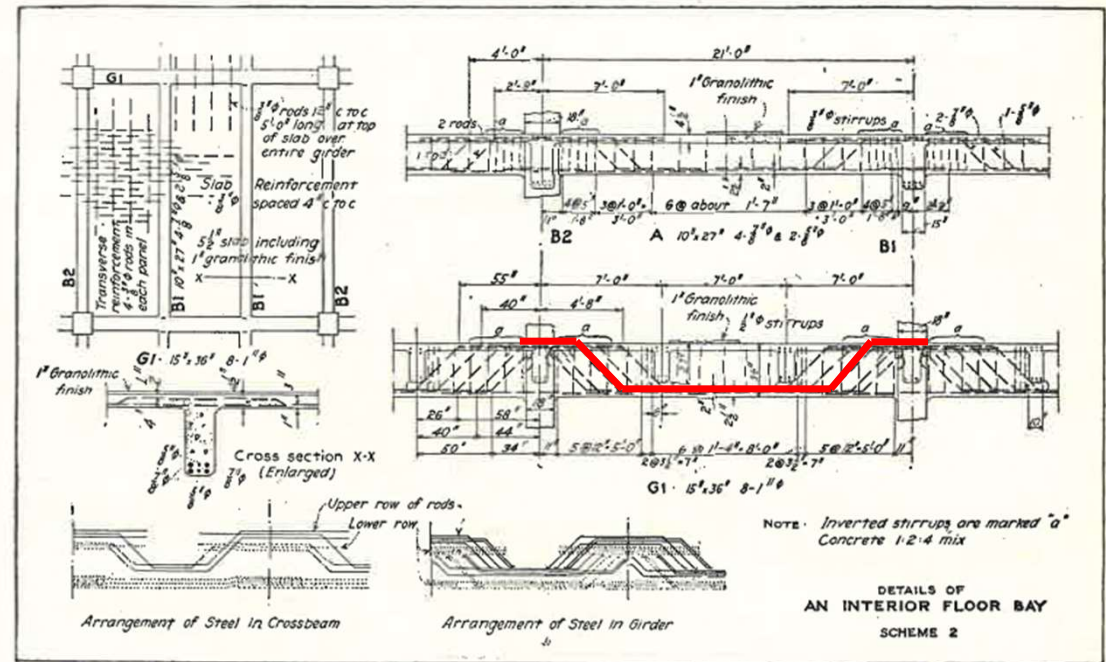
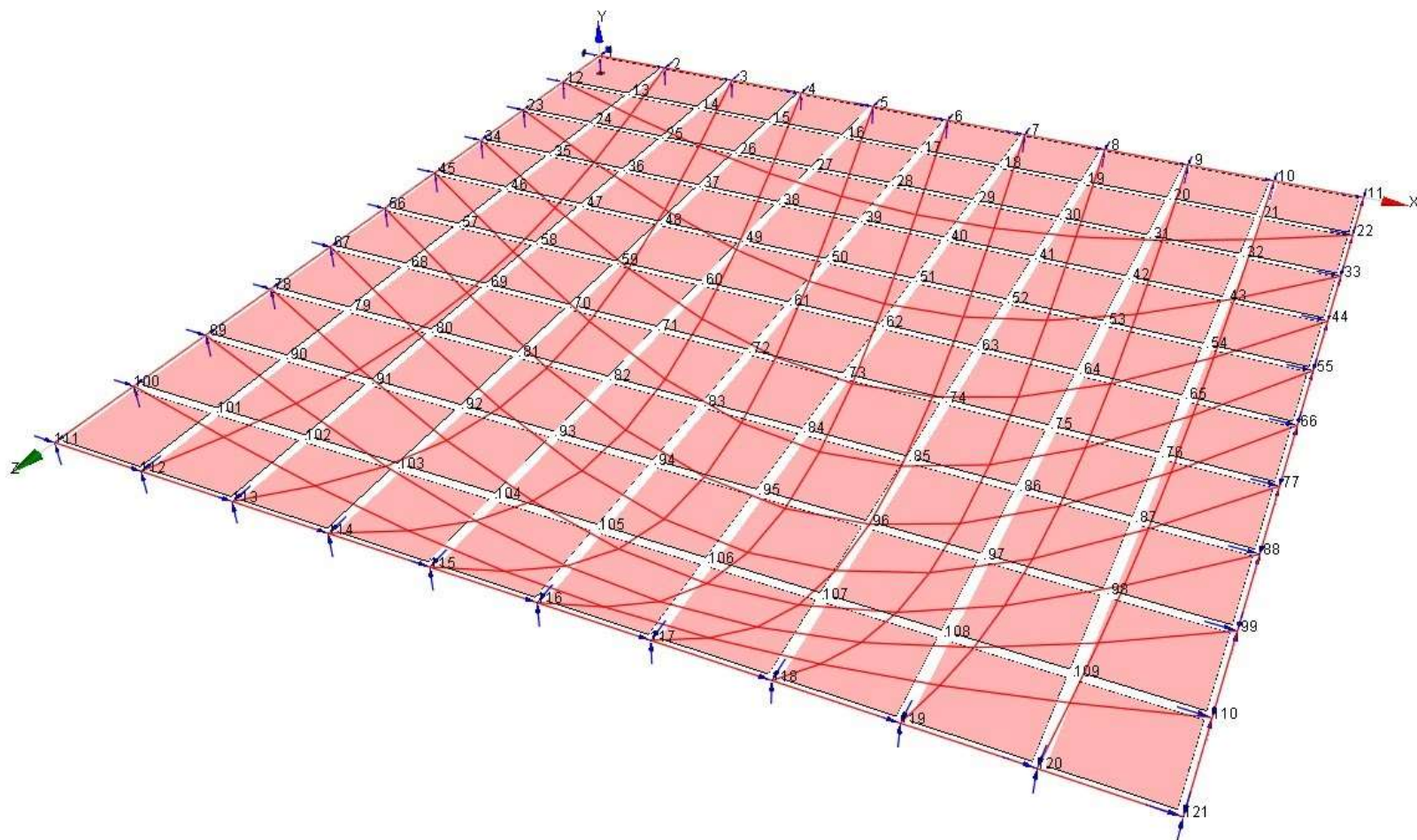
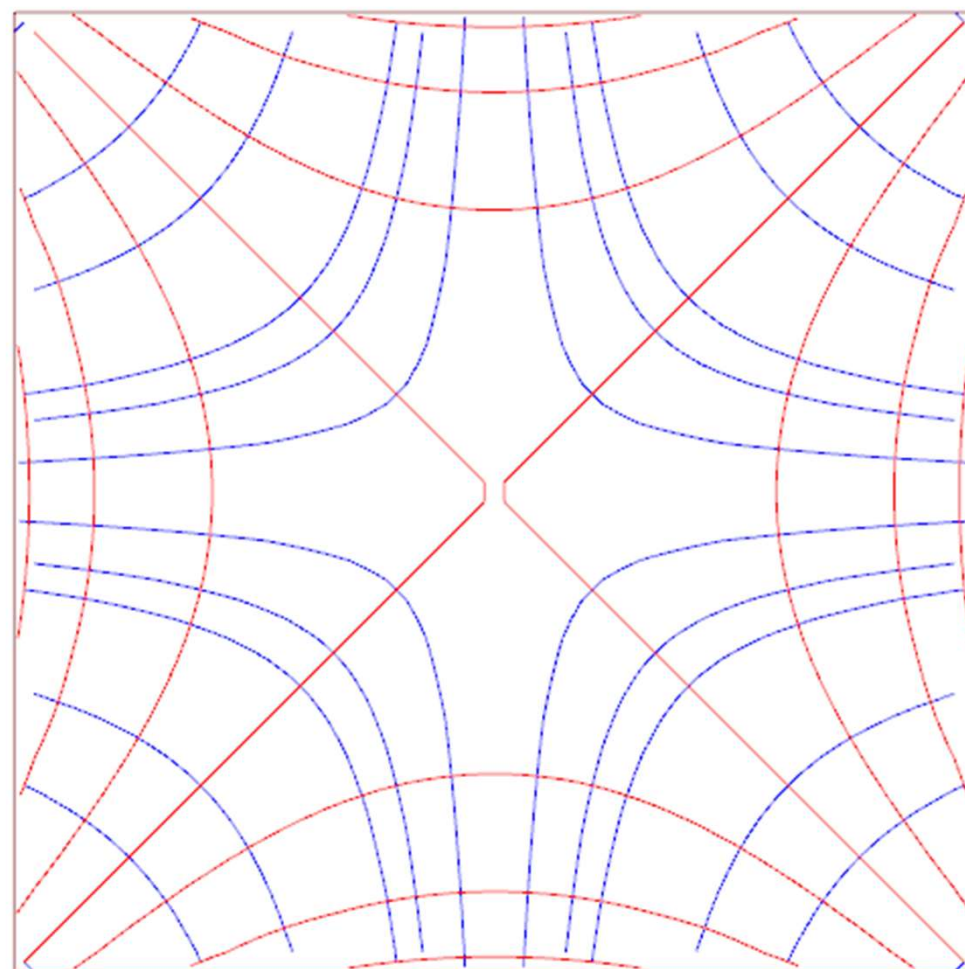


FIG. 4.

15. The common theory of flexure gives the unit stress correctly at the important section of maximum moment and also for the extreme fibers in other sections, since at these points the shear is zero. Where the shear is not zero an inclined stress is the result and the flexure formula gives only the horizontal component of this stress—namely, the fiber stress.







2

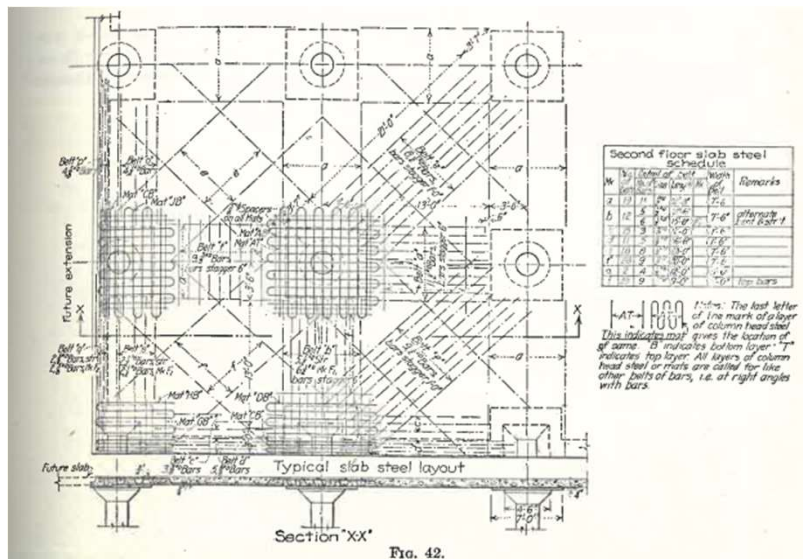


FIG. 42.

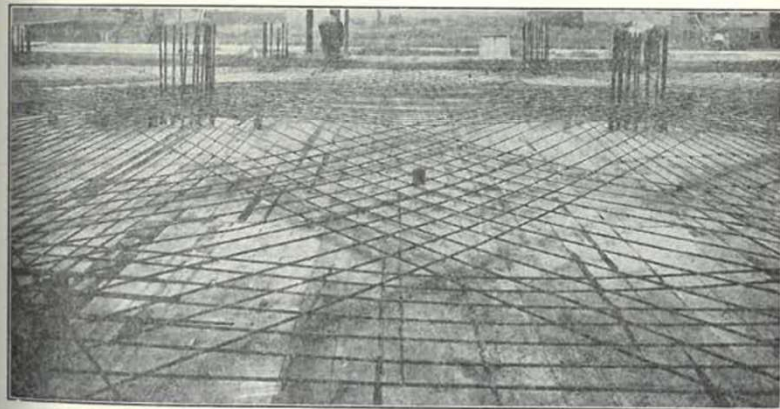


FIG. 43.

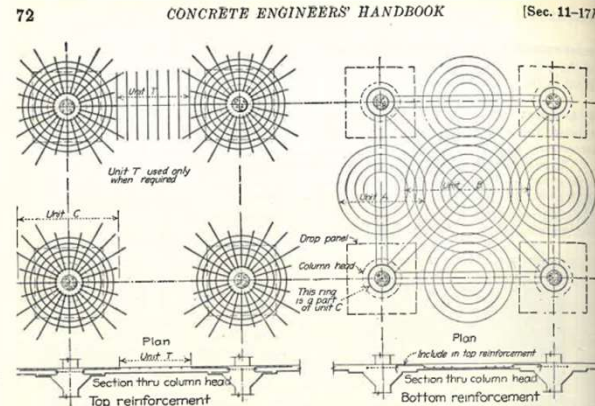


FIG. 50.

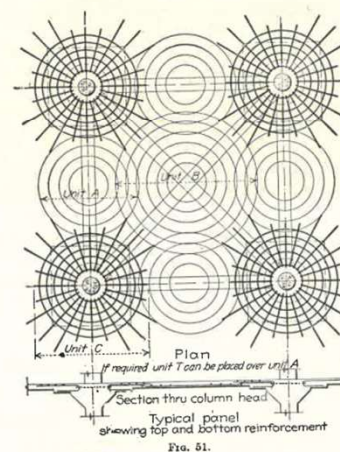


FIG. 51.

Sec. 11-17A]

BUILDINGS

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The trussed bar extends into the column head a sufficient distance beyond the point of maximum stress (i.e., the edge of the column head) to develop, in combination with the hook, their full tensile strength. Thering which they engage serves to distribute the bearing stresses laterally on to a large area of concrete.

Position of Units.—Units A and B are placed near the bottom while unit C is near the top of the slab.

Compression Reinforcement.—By introduction of compression reinforcement in the shape of lower prongs of the radials, the slab is stiffened at the support, and the compression stresses in concrete reduced. If desired, therefore, it is possible to omit the drop panel at the column head and use an altogether flat ceiling. This is often desirable either for the sake of appearance or to simplify shafting or piping.

Secondary Reinforcement.—Sometimes to prevent cracks on the top of the slab between columns, additional secondary reinforcement consisting of short straight bars, and called Unit T, is used. These bars are usually placed after the concrete of the slab is poured.

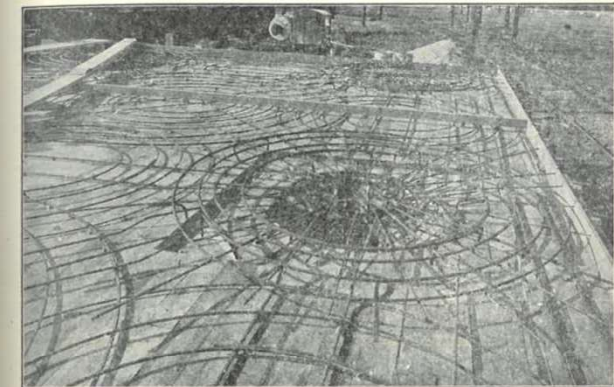


FIG. 52.

THEORETICAL DISCUSSION

The scientific basis of the S-M-I system is evident from the following discussion of the action of a flat slab under load.

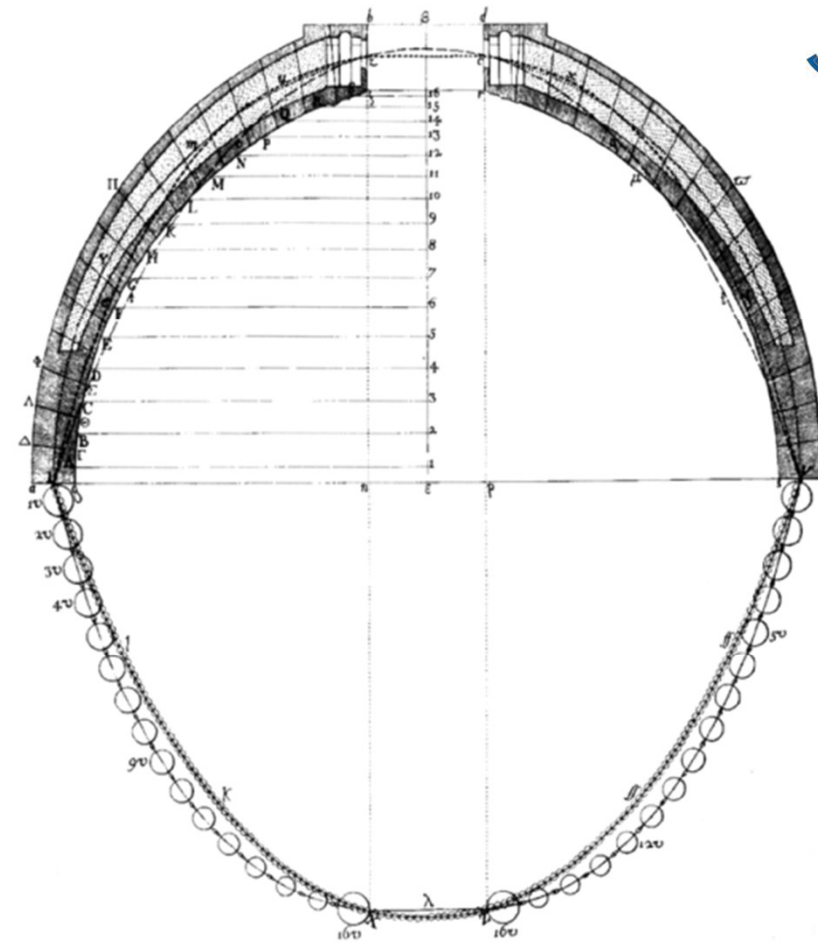
Shape of the Slab after Deflection.—After deflection a flat slab assumes a composite shape, namely, the shape of an umbrella at the column head, and the shape of a saucer in the central portion.

Lines of Equal Deflection.—The shape of a deflected slab can be seen better from Fig. 53, which shows in sections the deflection curve along the side and the diagonal of the panel, and in plan the lines of equal deflection. The lines of equal deflection, which are based on tests, were obtained by connecting the points which deflected an equal distance below their original position.

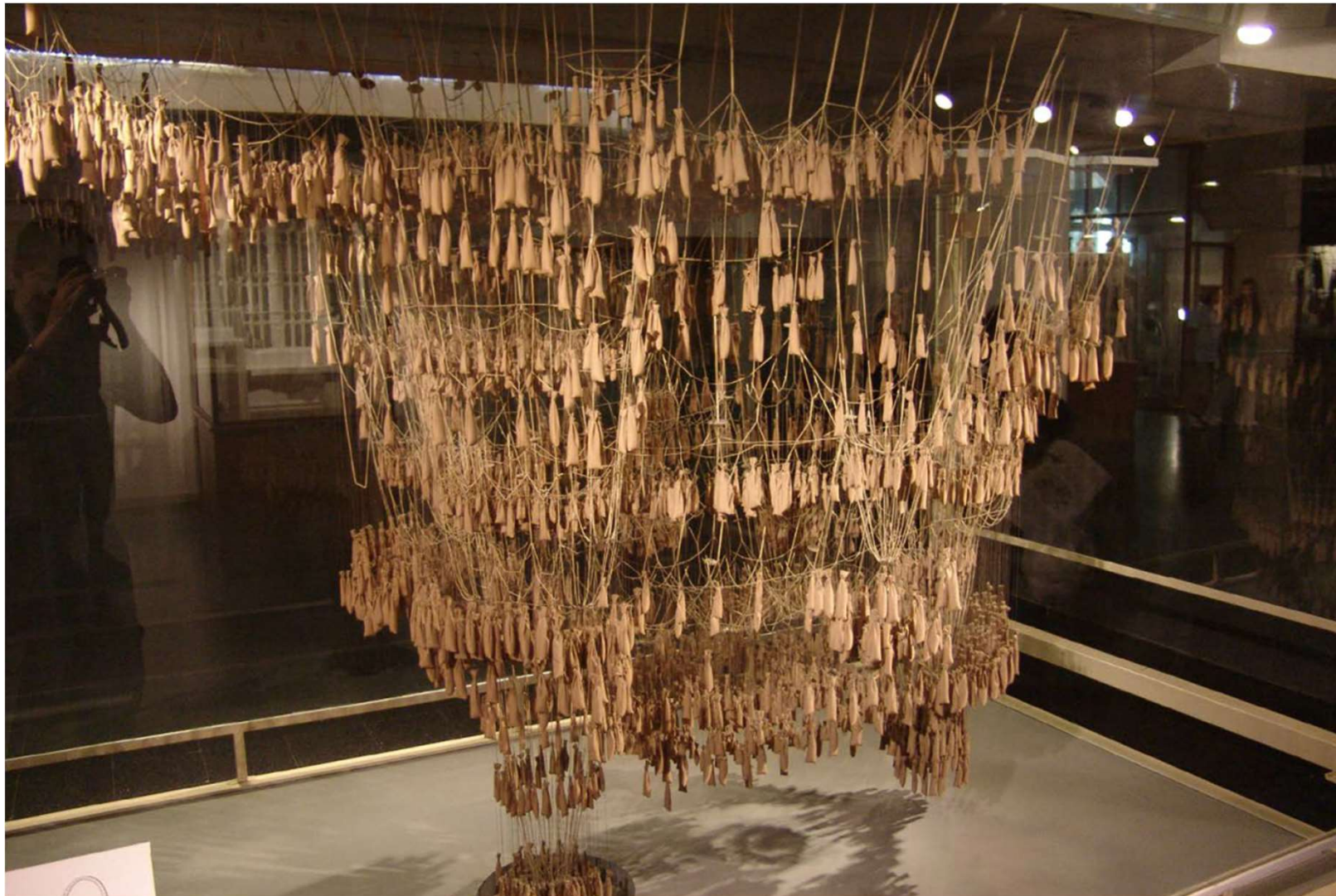
Direction of Stresses and Reinforcement.—By referring to the plan and the sections, it is evident that deformation of fibers are equal and therefore the fiber stresses act perpendicularly to the lines of equal deflection as indicated by arrows in Fig. 53. The best method of resisting these stresses, or preventing the deformation, is either by placing the bars perpendicularly to the lines of equal deflection, or by enclosing them by means of a ring, the hooping action of which is explained later. Fig. 54 shows the deflection lines in light dash lines and the reinforcement according to the S-M-I system in heavy lines. The radials and trussed bars are perpendicular to the lines of equal deflection. The rings either intersect the deflection lines at angles close to 90 deg., or they enclose the same and prevent the enclosed concrete from spreading. The combination therefore fulfills all the requirements of efficient and economical reinforcement.



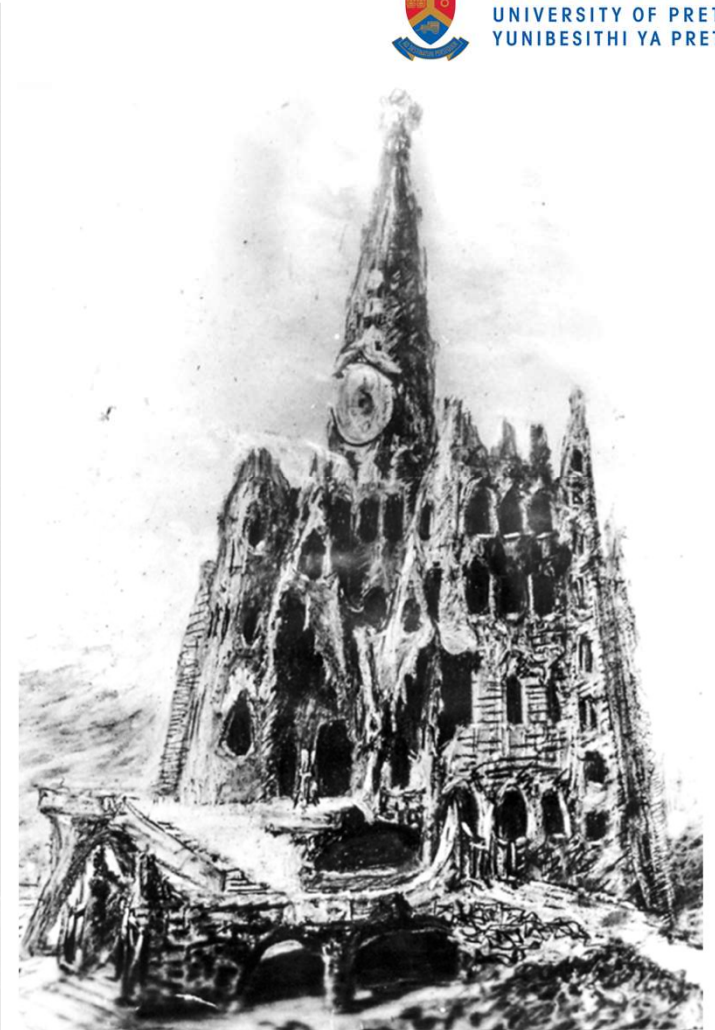
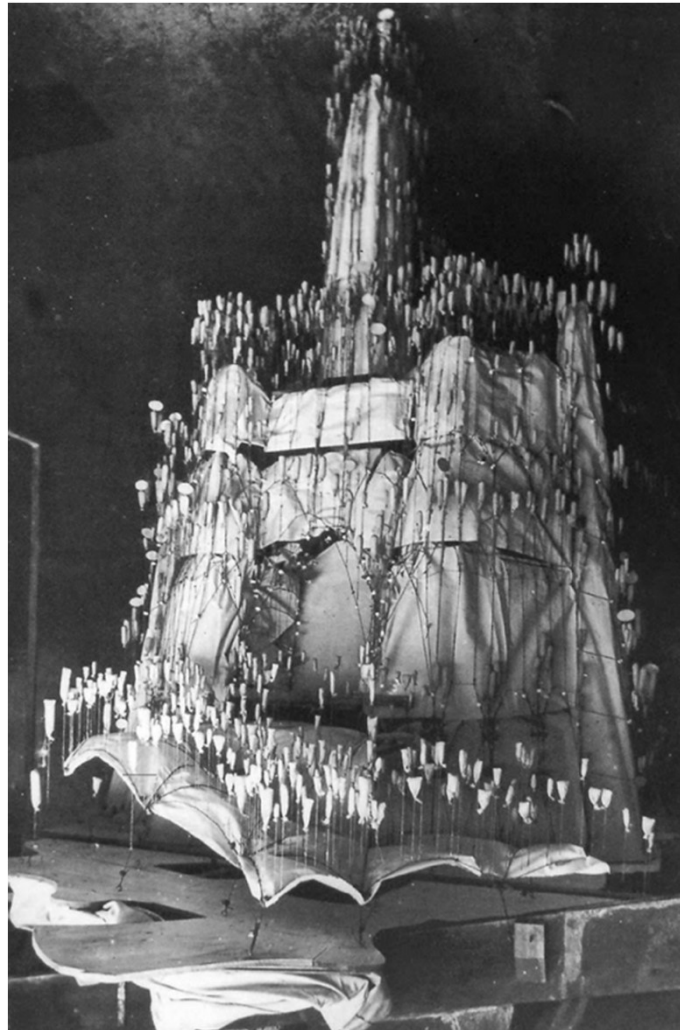
Gatti Wool Factory, Rome, 1951 | Pier Luigi Nervi



Giovanni Poleni, 1748: Hooke's guiding principles

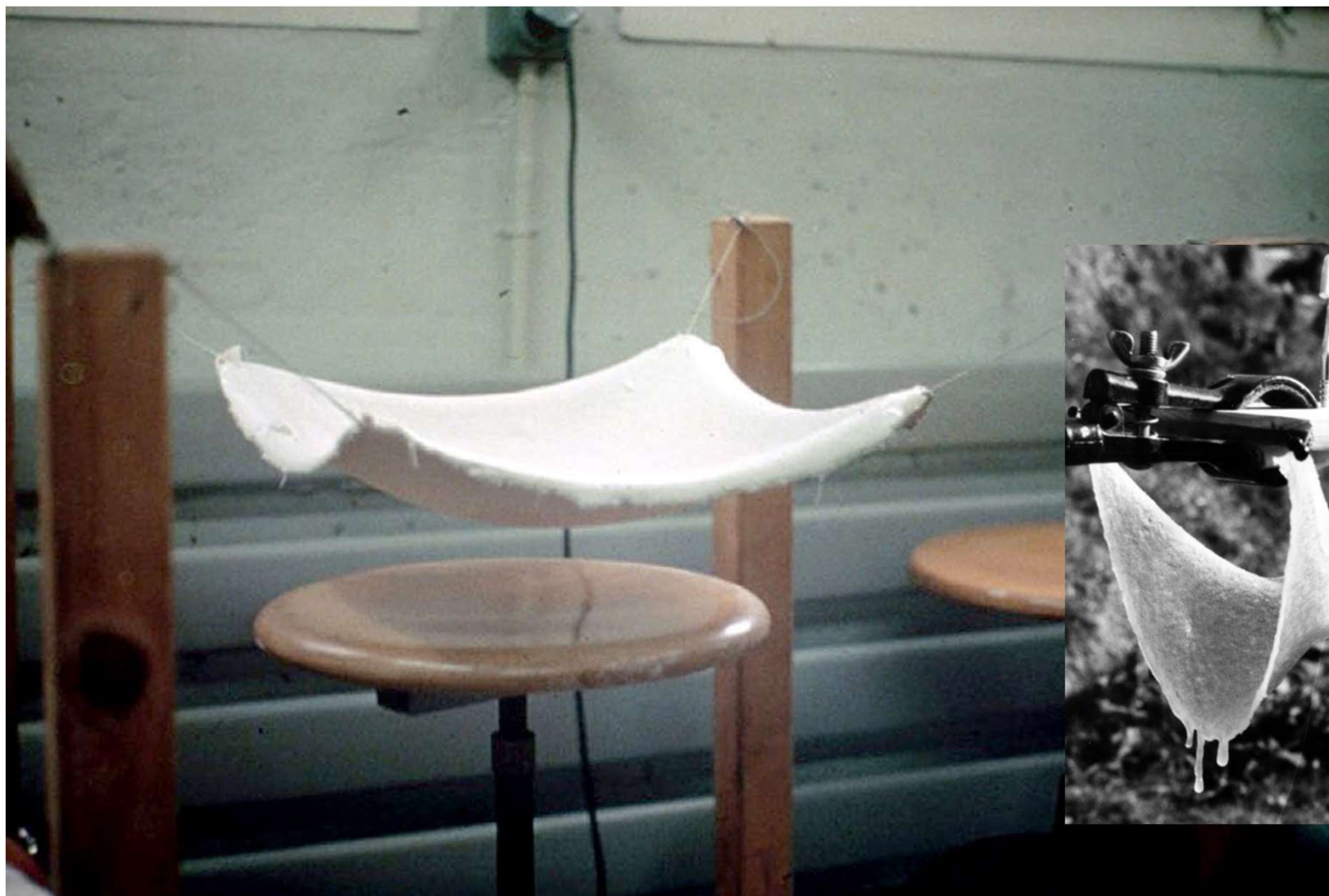


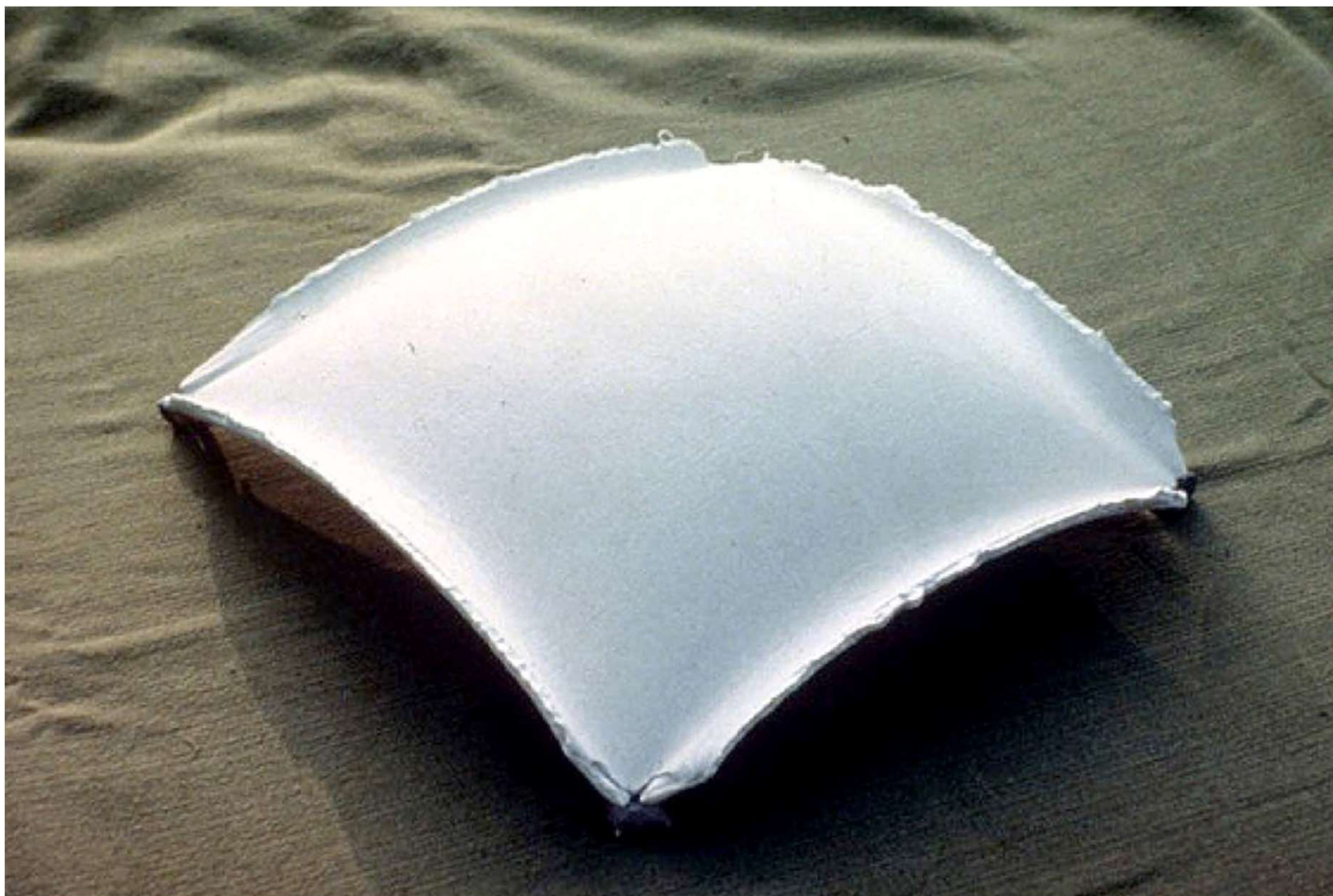
Model of Familia Sagrada, Barcelona, 1908 | Antoni Gaudí



Model of Familia Sagrada, Barcelona, 1908 | Antoni Gaudi



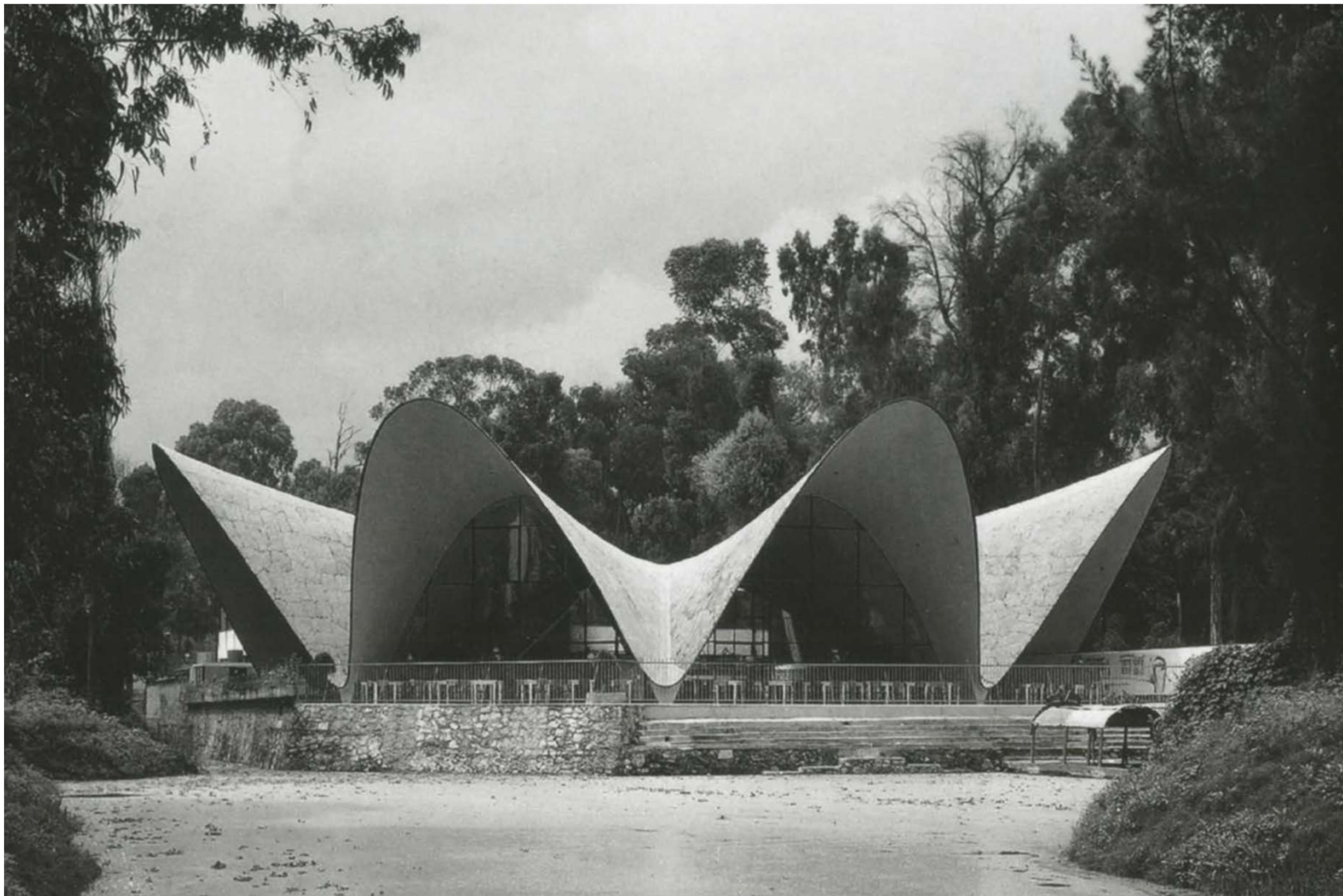




Heinz Isler



Filling station, Deitingen Süd, 1968 | Heinz Isler

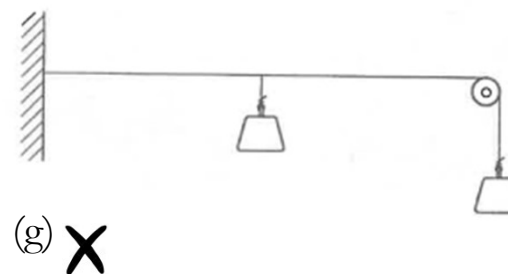
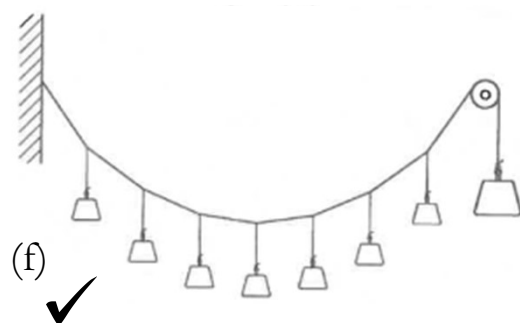
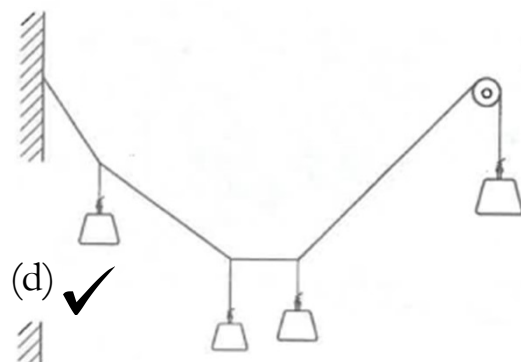
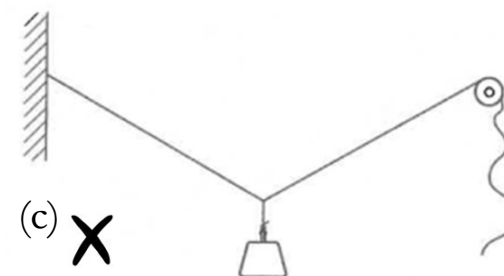
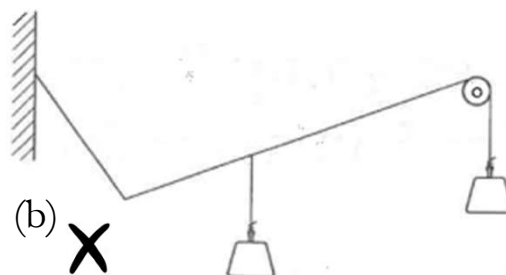
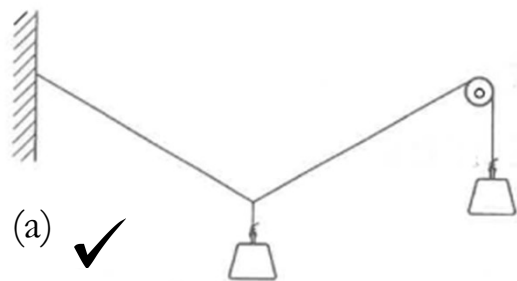


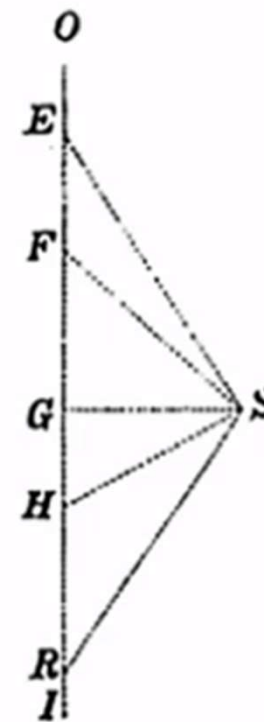
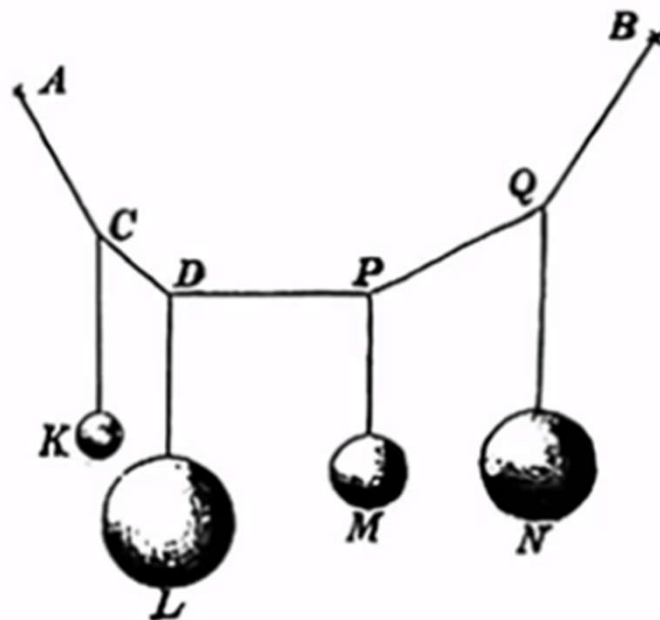
Los Manantinales Restaurant, Xochimilco, 1958 | Felix Candela



Cadyl Horizontal Silo, Young, 1978 | Eladio Dieste

Graphic statics







Carl Culmann 1821-1881

GRAPHISCHE STATIK

C. CULMANN.

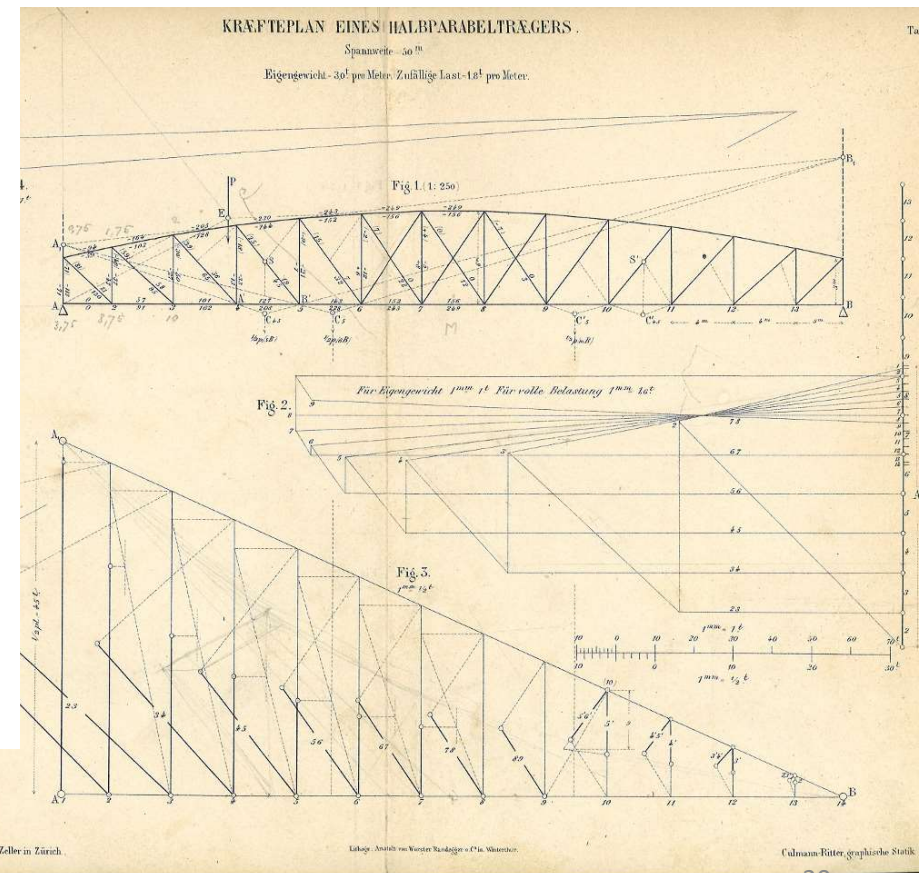
ZWEITE VON BEKANNTESTE AUFLAGE.

ERSTER BAND.

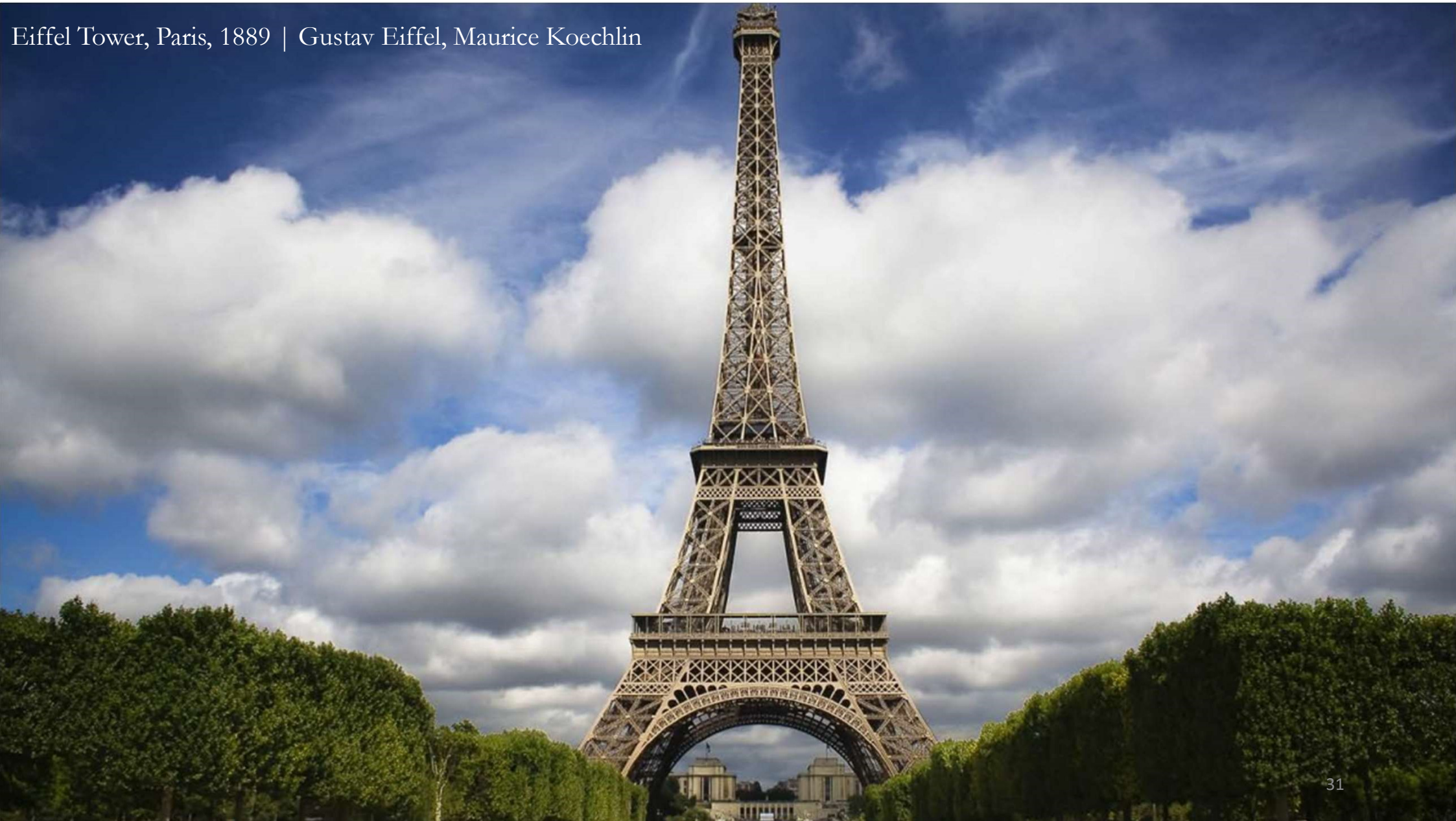
Mit 100 in das Text gedruckten Holzschnitten und 10 Tafeln.

ZÜRICH

VERLAG VON MEYER & ZELLER
(A. Reimann)
1871.



Eiffel Tower, Paris, 1889 | Gustav Eiffel, Maurice Koechlin



Epure de résistance au vent

- 1^{er} Cas : Vent de 800 m de la base au sommet.
2^e Cas : Vent croissant de 200 m de la base jusqu'à 800 m au sommet.

Surfaces et efforts correspondants.

Traverse	Hauteur	Surface	Effort	Effort	Effort
			du vent	du vent	du vent
1	787	300	300	300	300
2	1046	300	300	300	300
3	1815	300	300	300	300
4	2584	300	300	300	300
5	3353	300	300	300	300
6	4122	300	300	300	300
7	4891	300	300	300	300
8	5660	300	300	300	300
	300				

Détermination des efforts dans les arbalétriers.

Le prolongement du montant coupe par la section AB rencontre l'axe au point O où agit la résultante des forces 1, 2, 3, 4, 5. On peut donc en ce point O décomposer cette force de 1.807.200 suivant la direction des montants.

Ce qui donne pour chacun d'eux un effort de 3.600.000.

De même :

L'effort à la partie inférieure d'un montant est 3.600.000.

L'effort à la partie supérieure d'un montant est 3.600.000.

Calcul de la section d'un montant à sa base.

Poids total de la construction au niveau des appuis 5.600.000.

Moment de renversement à la base 503.150.120.

Charge à la base d'un montant due au poids propre 1.675.000.

Charge à la base d'un montant due à l'effet du vent 1.675.750.

Charge totale 3.350.750.

Section d'une membrure à sa base 30.165 m².

Section d'un montant : 80.145 m².

Coefficient de travail à la base 5.199.280 = 9^e par m².

Polygone des forces du 1^{er} cas de surcharge due au vent.

Polygone des forces du 2^e cas de surcharge due au vent.

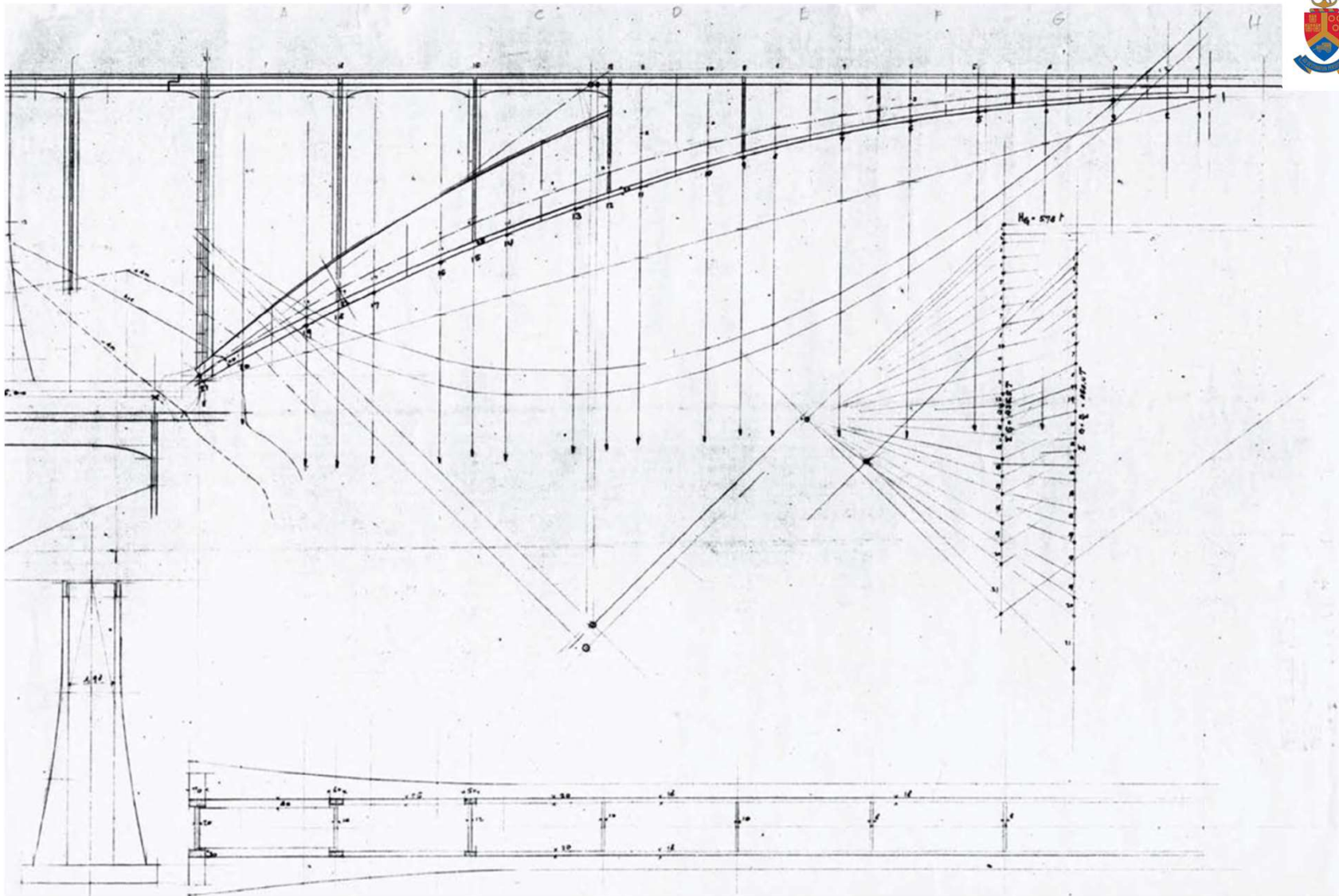
Echelle des forces : 1 cm pour 10.000.



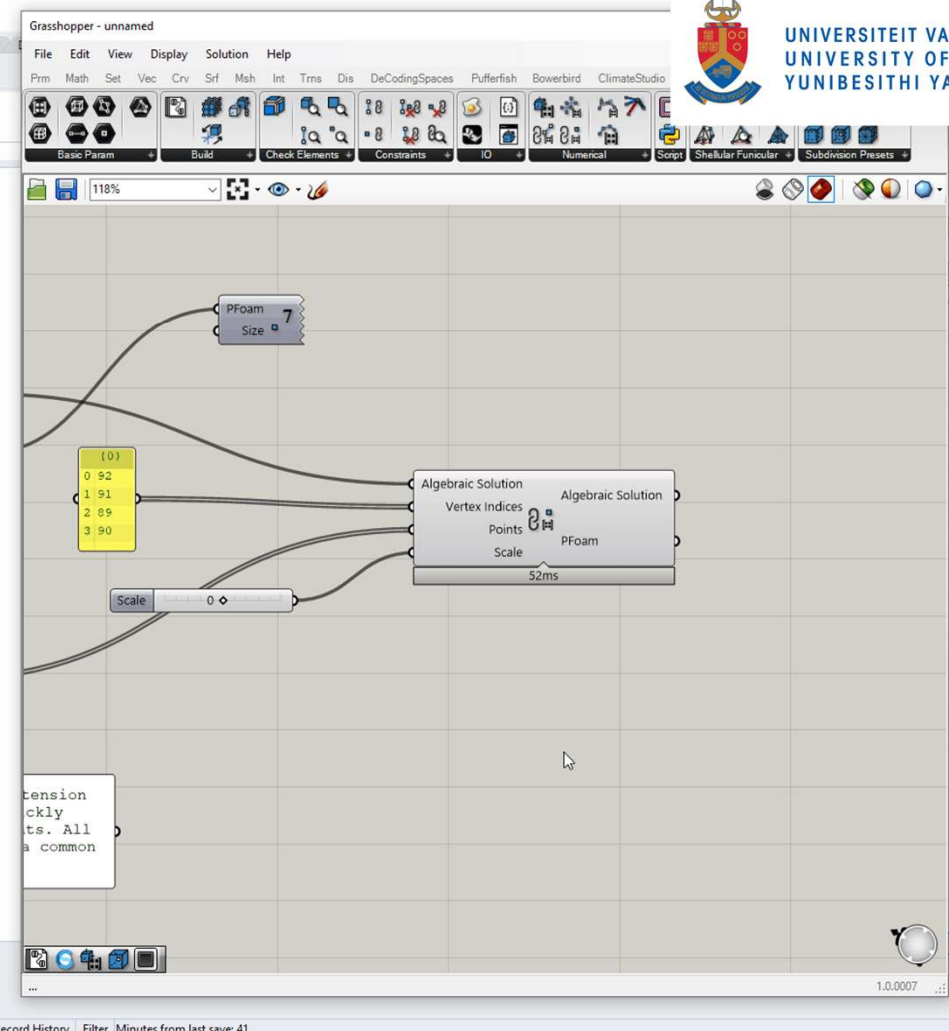
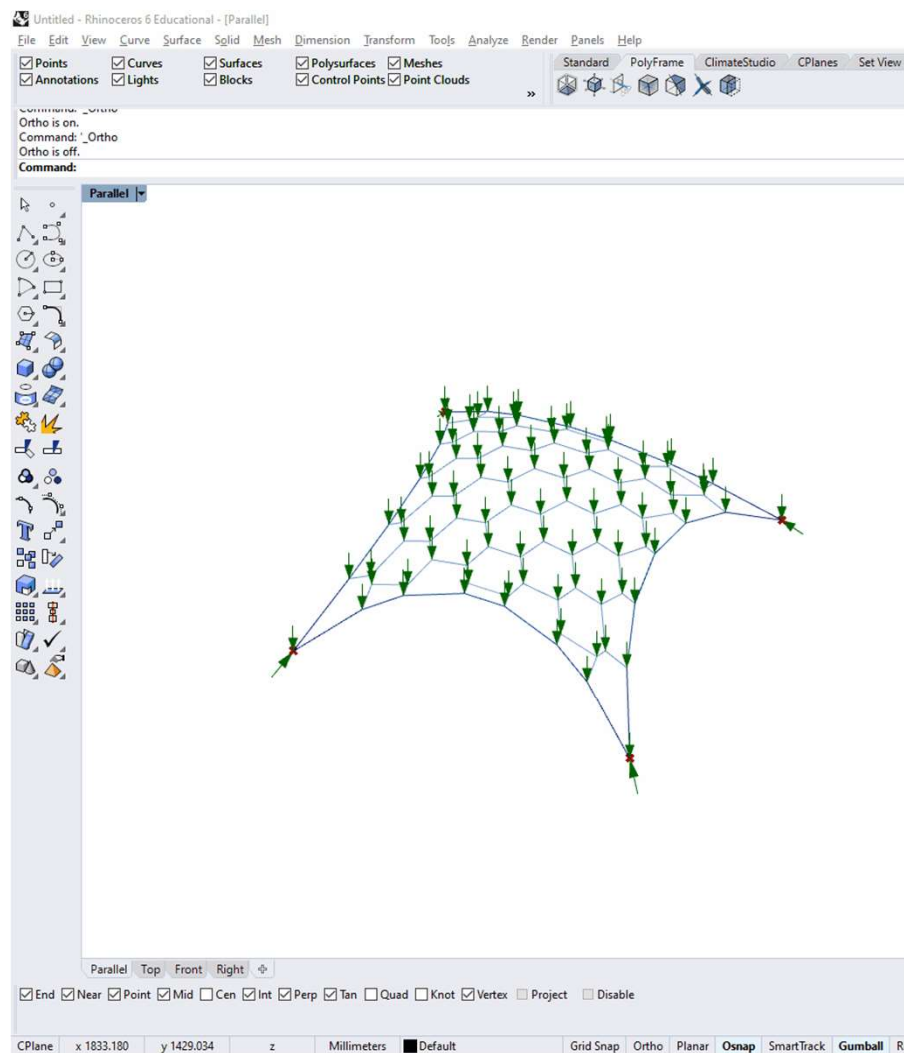
Garabit Viaduct, Ruynes, 1884 | Gustav Eiffel, Maurice Koechlin, Léon Boyer



Salginatobel Bridge, Schiers, 1930 | Robert Maillart



Salginatobel Bridge, Schiers, 1930 | Robert Maillart



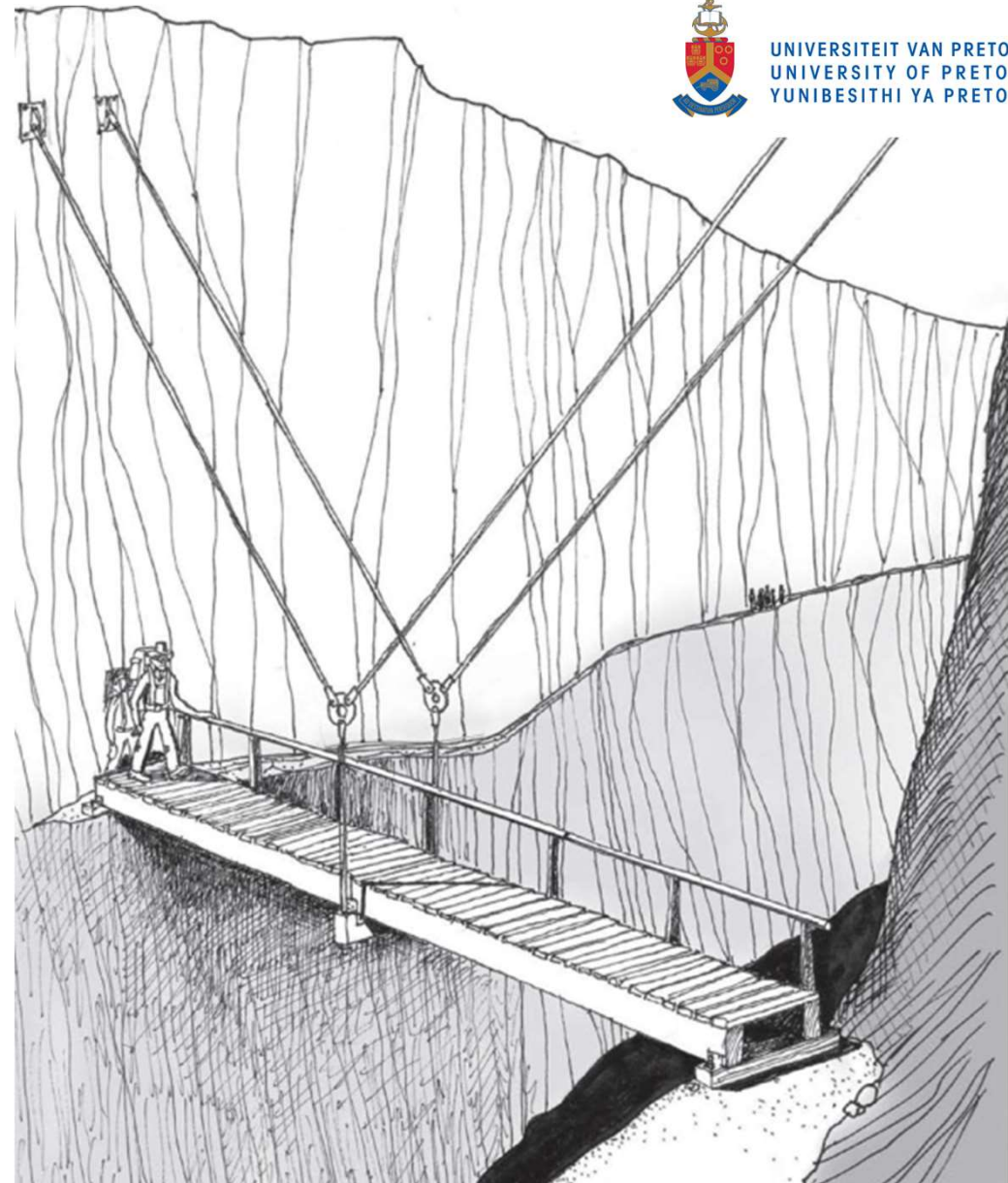
Example

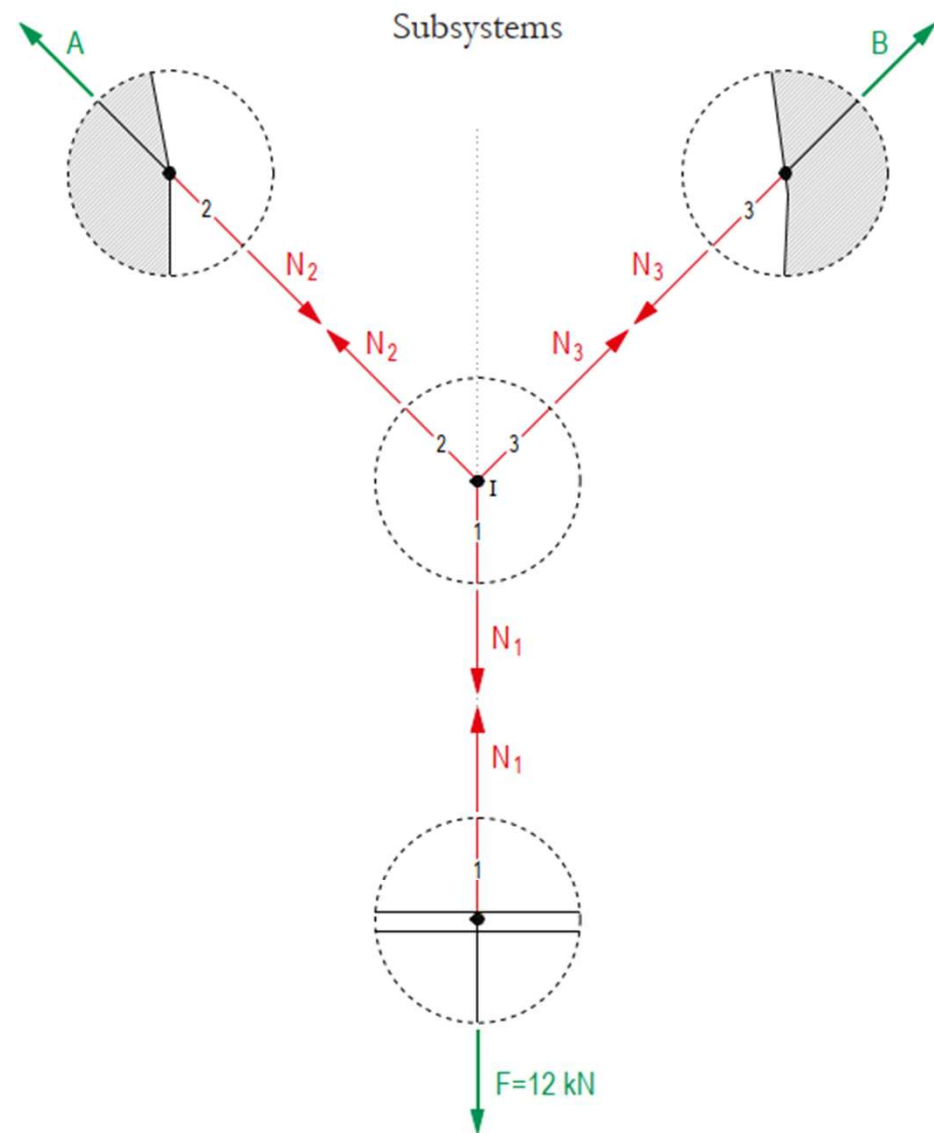
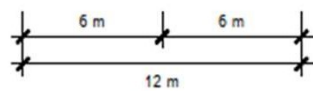
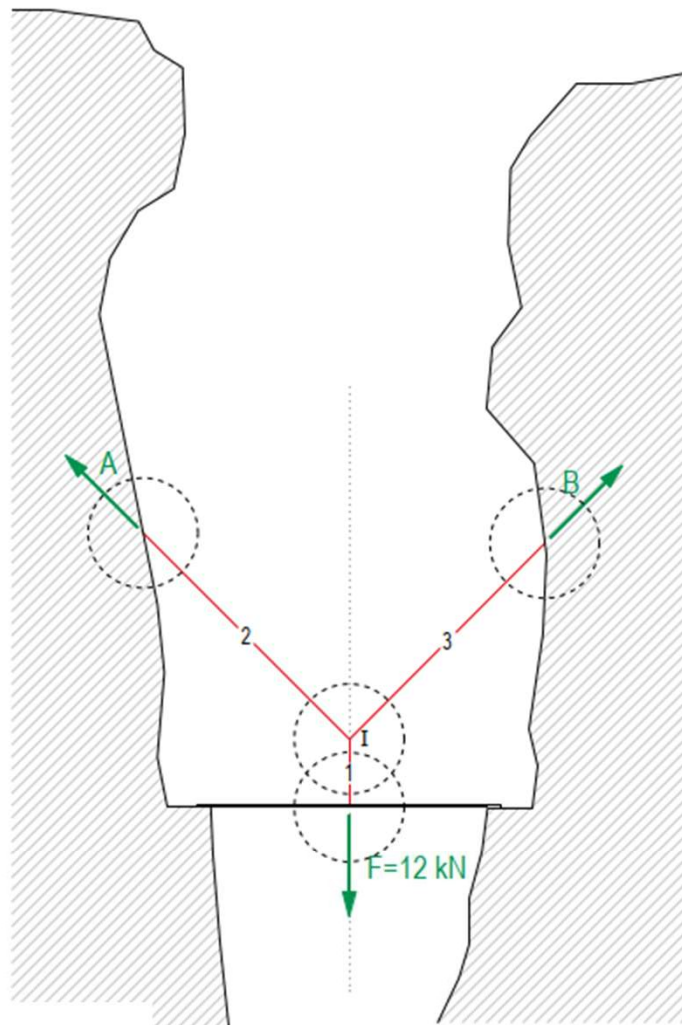


BRG
ETH zürich

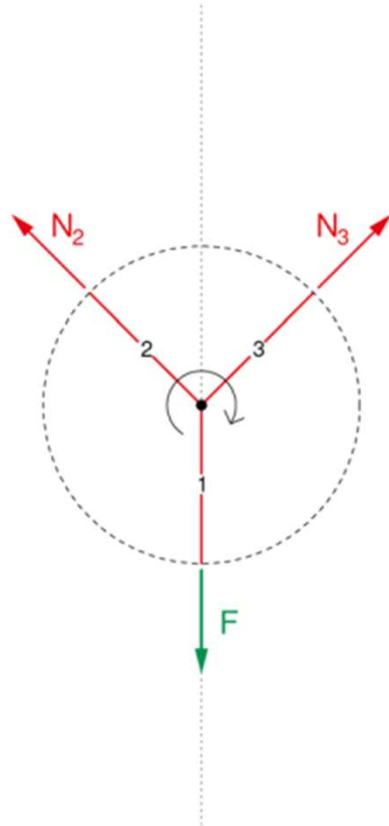


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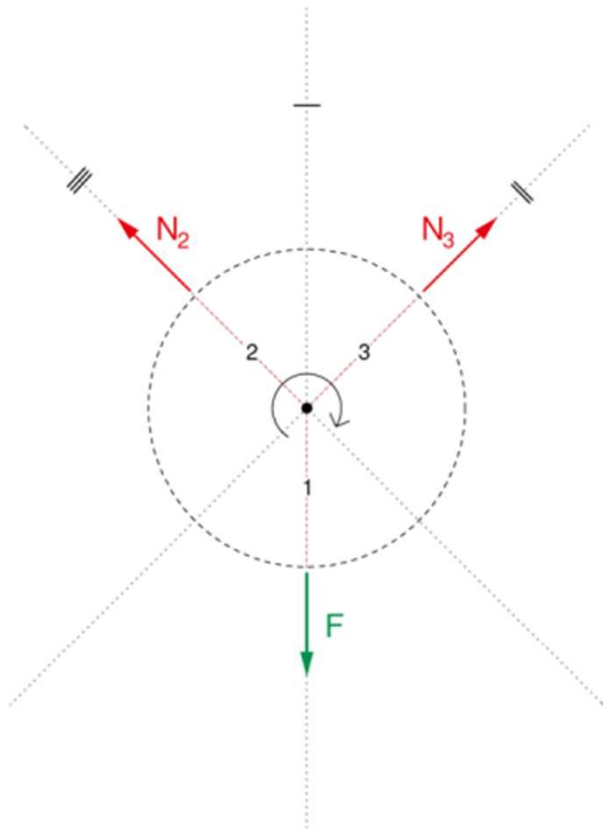
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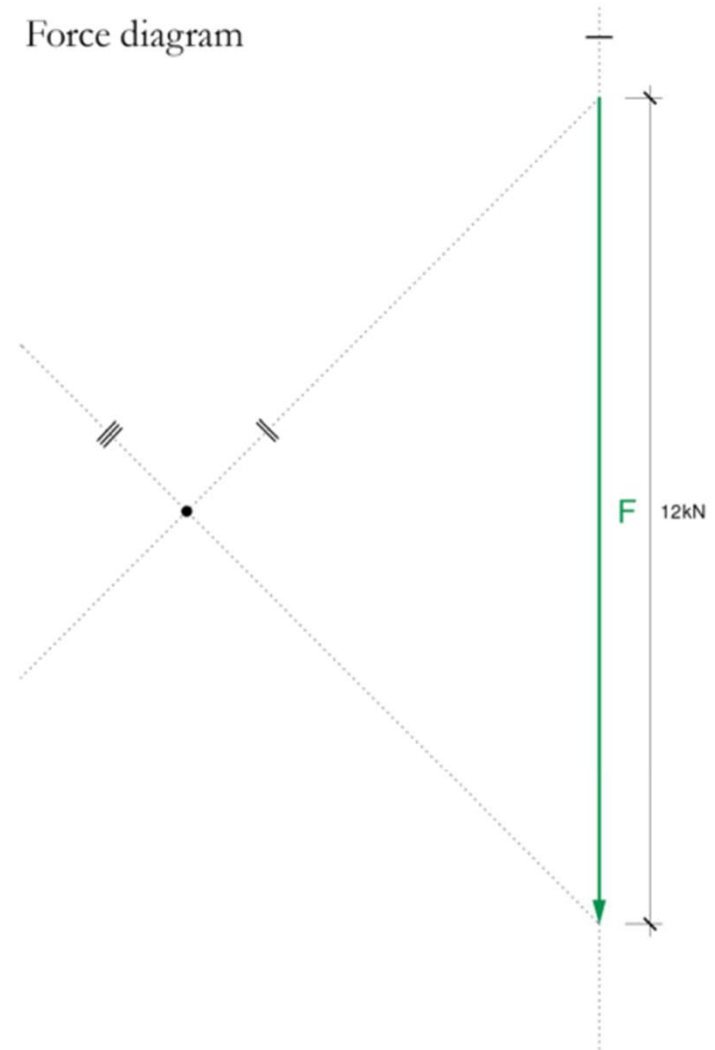
Force diagram



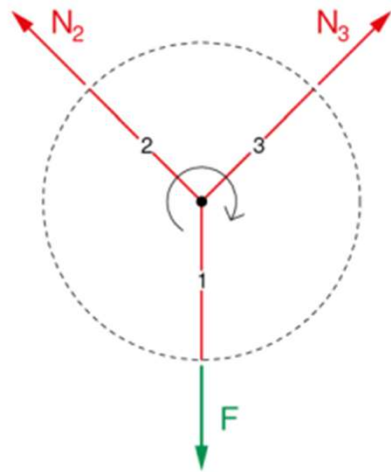
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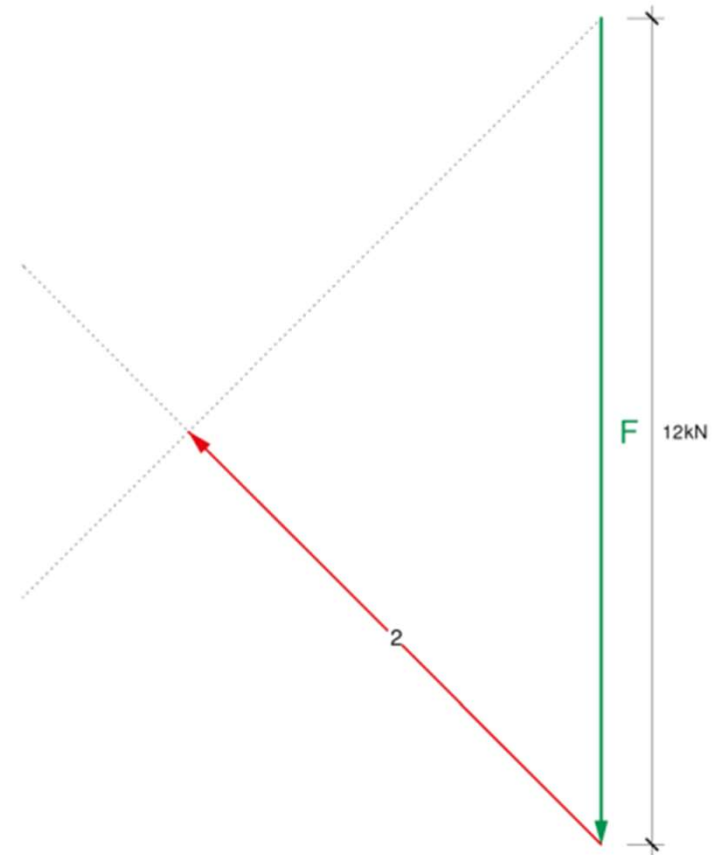
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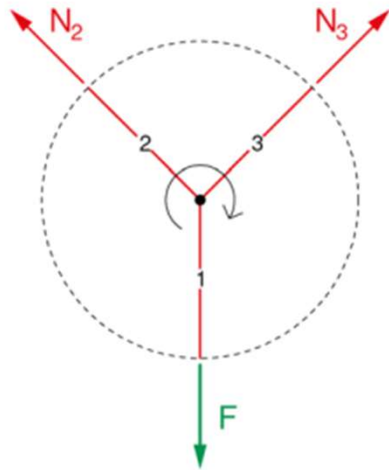
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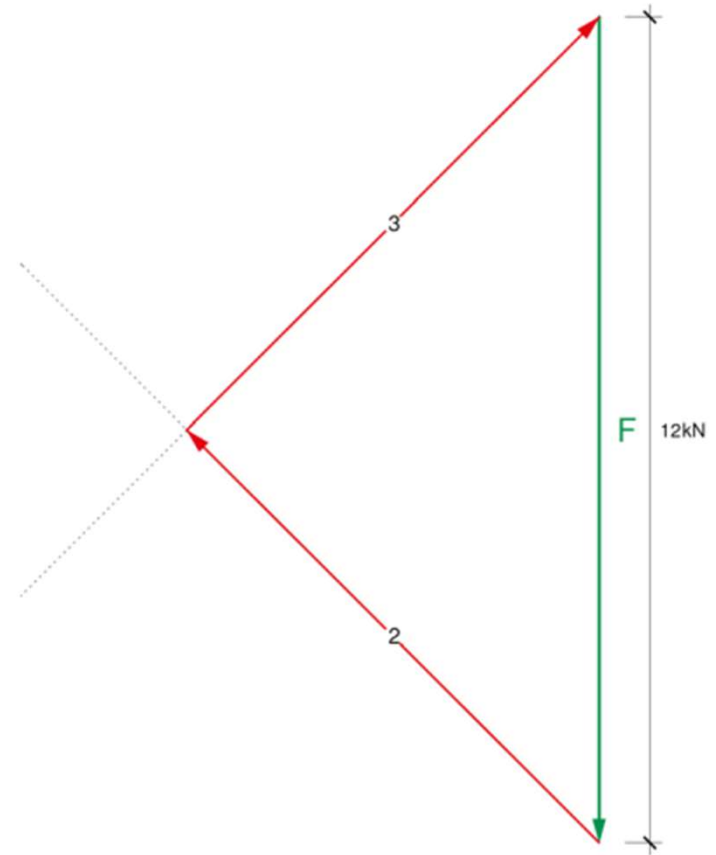
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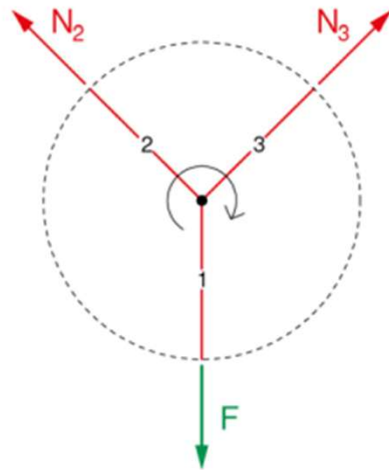
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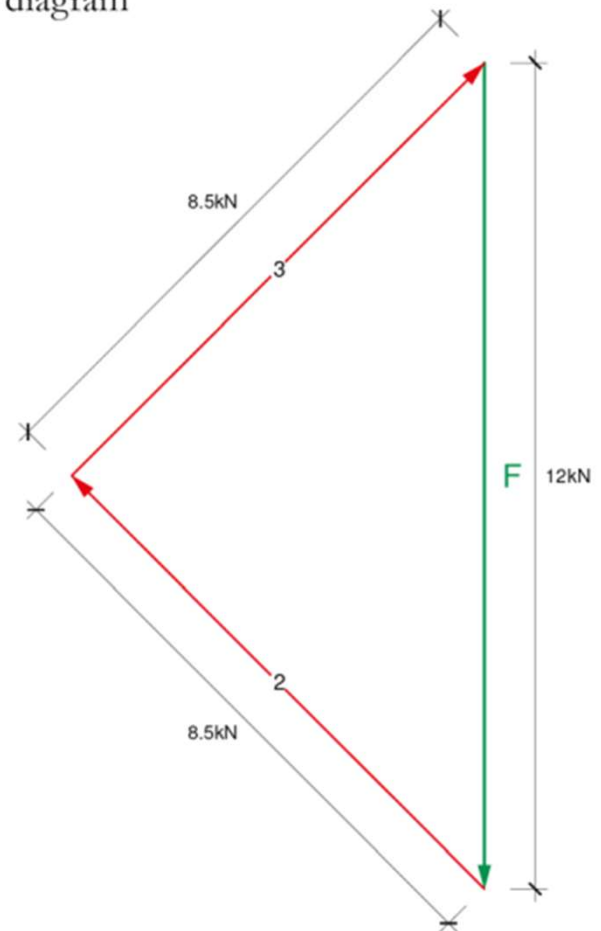
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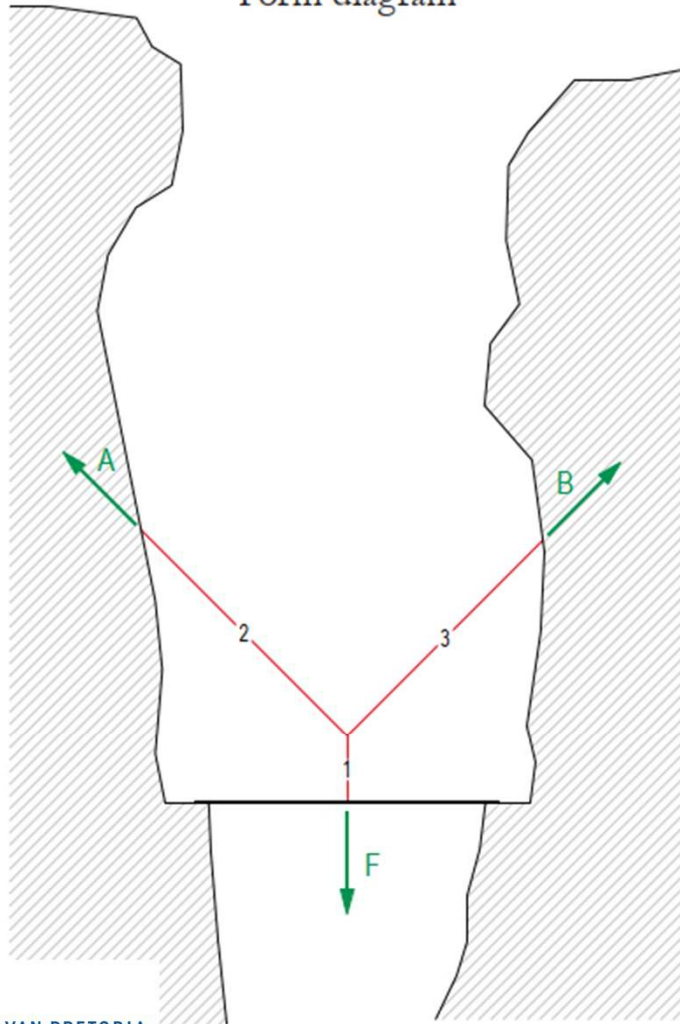
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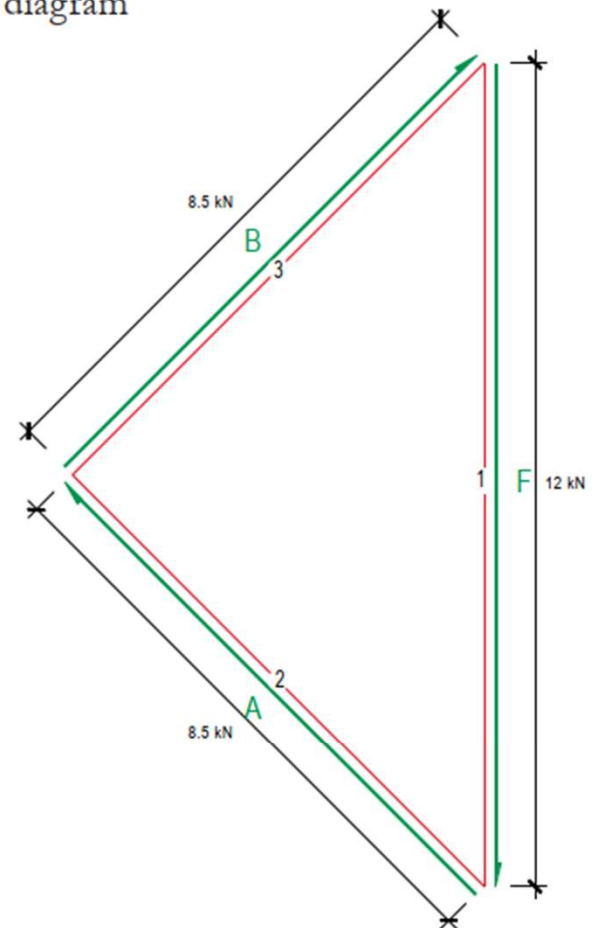
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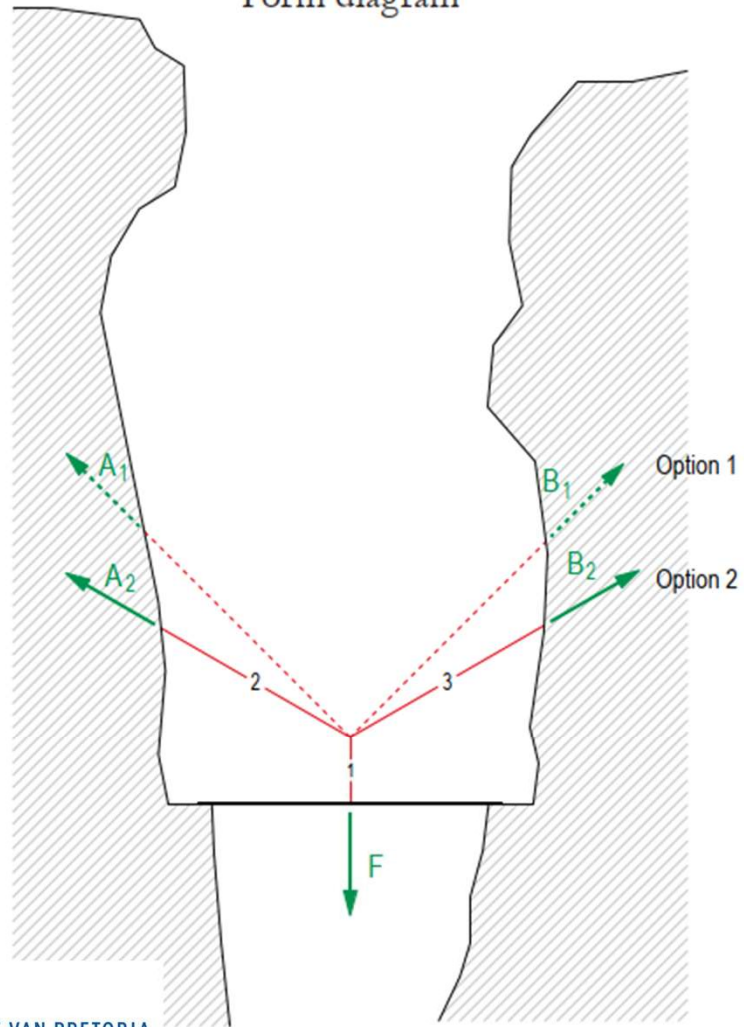
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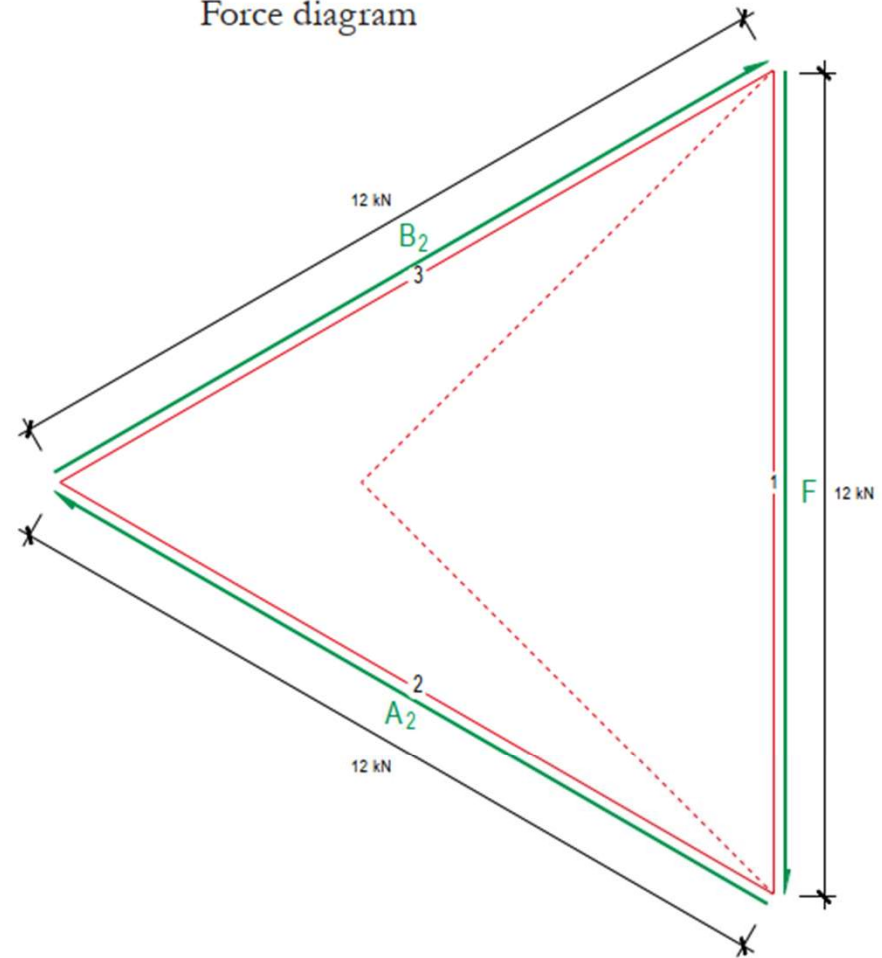
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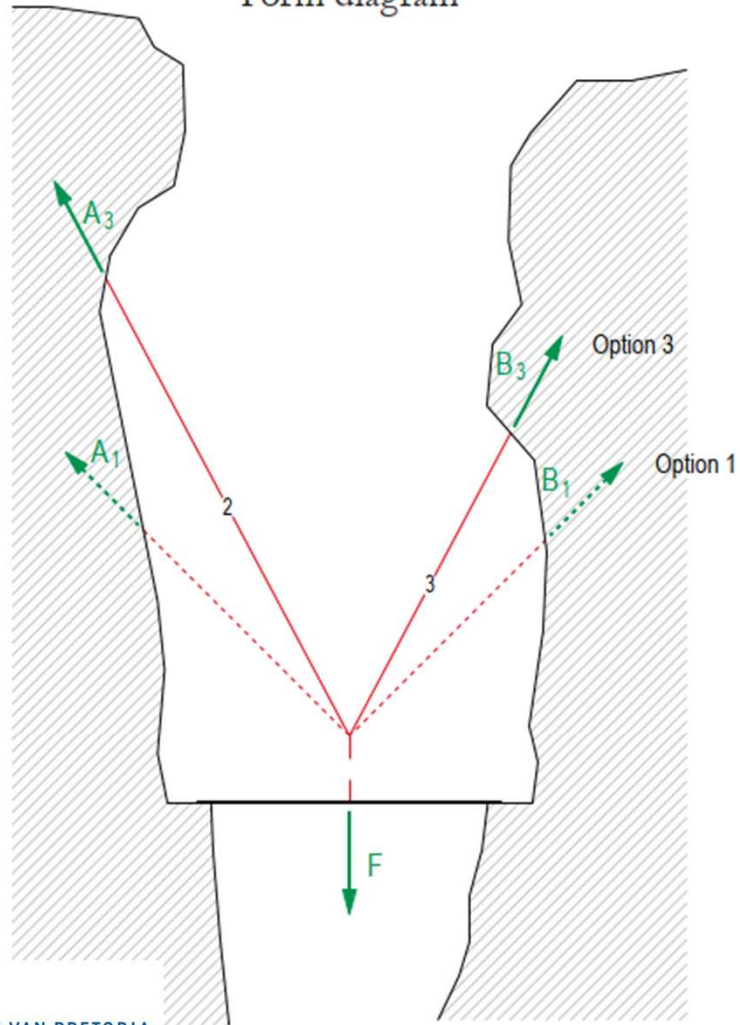
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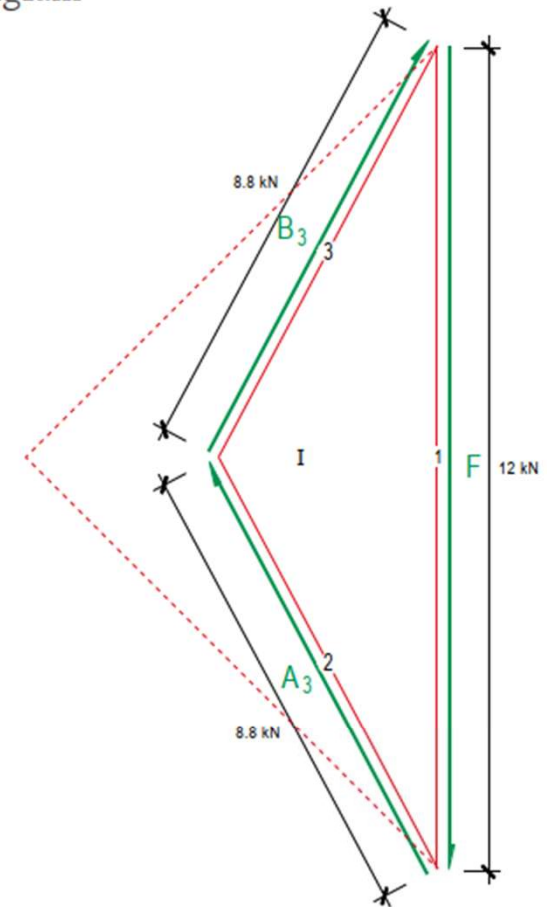
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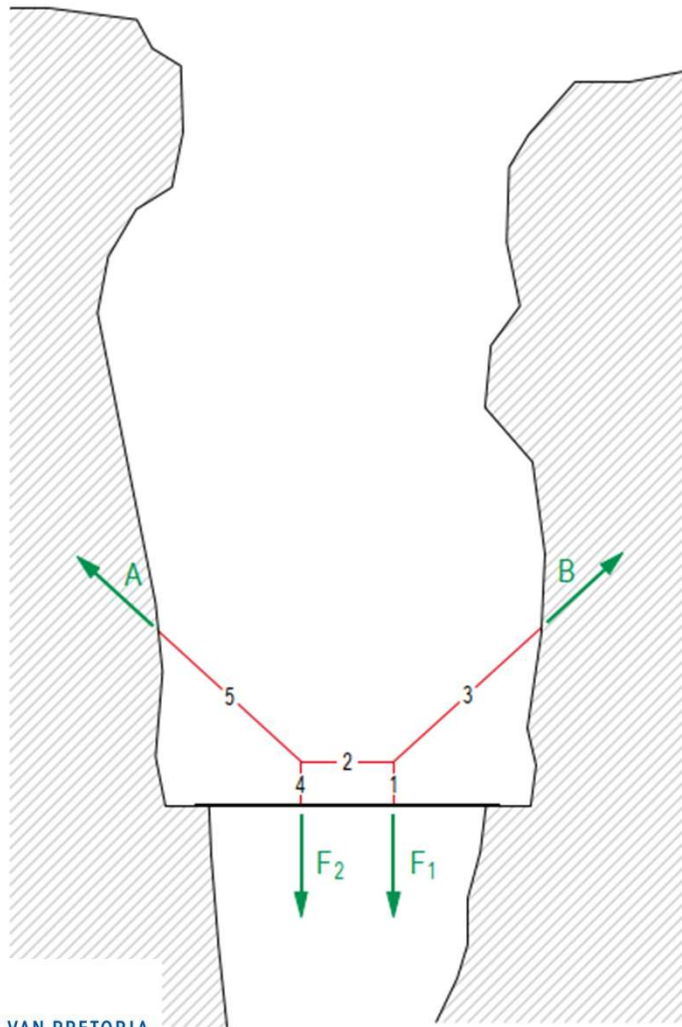


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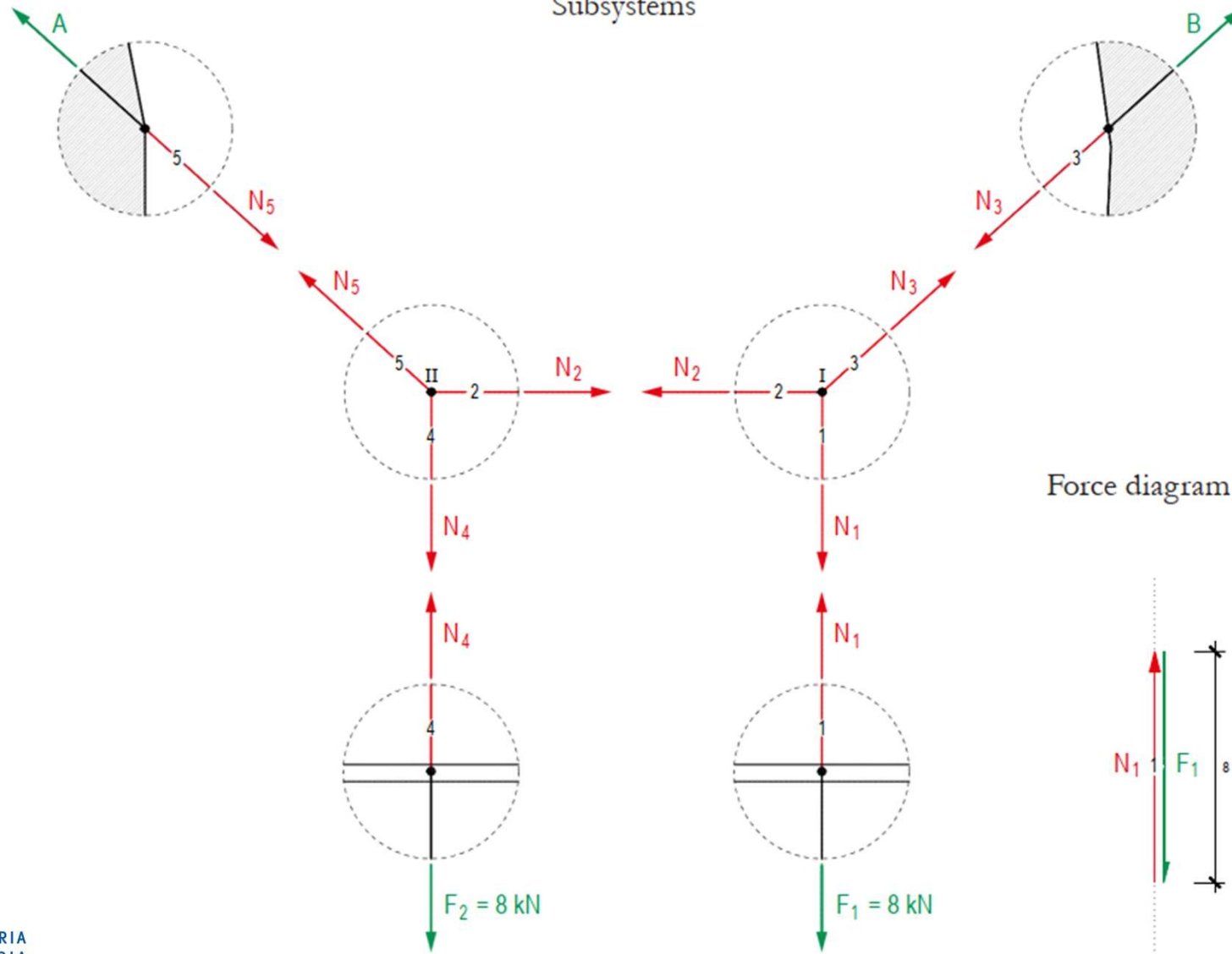


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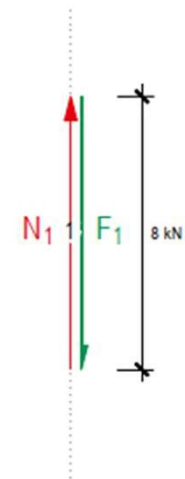




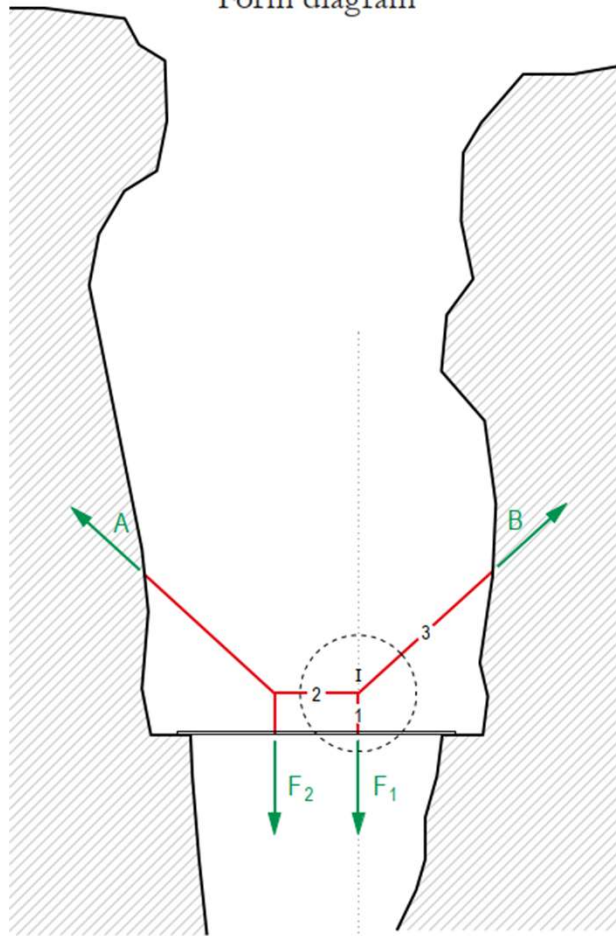
Subsystems



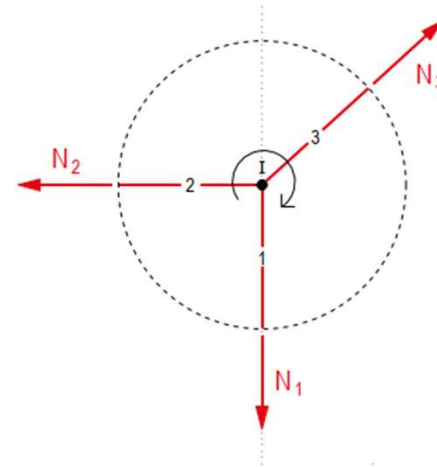
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Form diagram



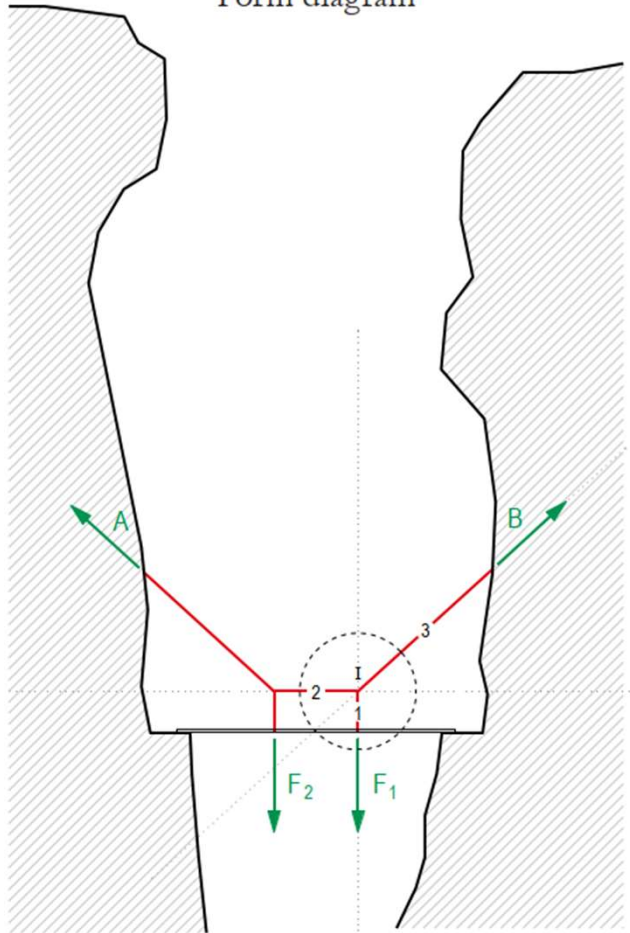
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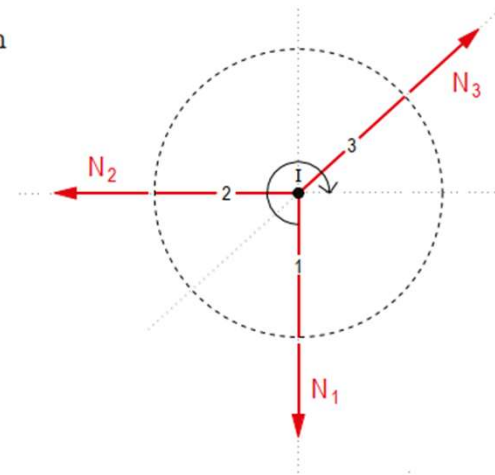
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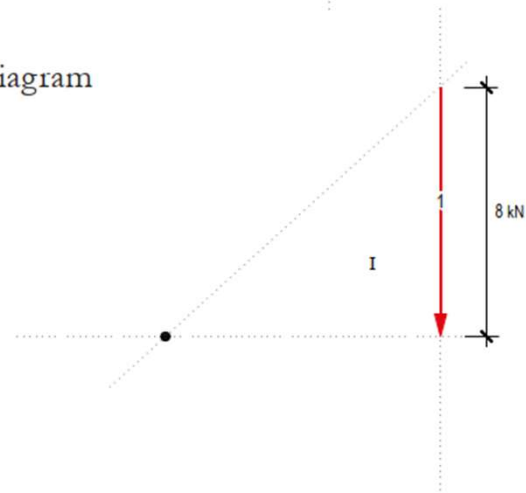
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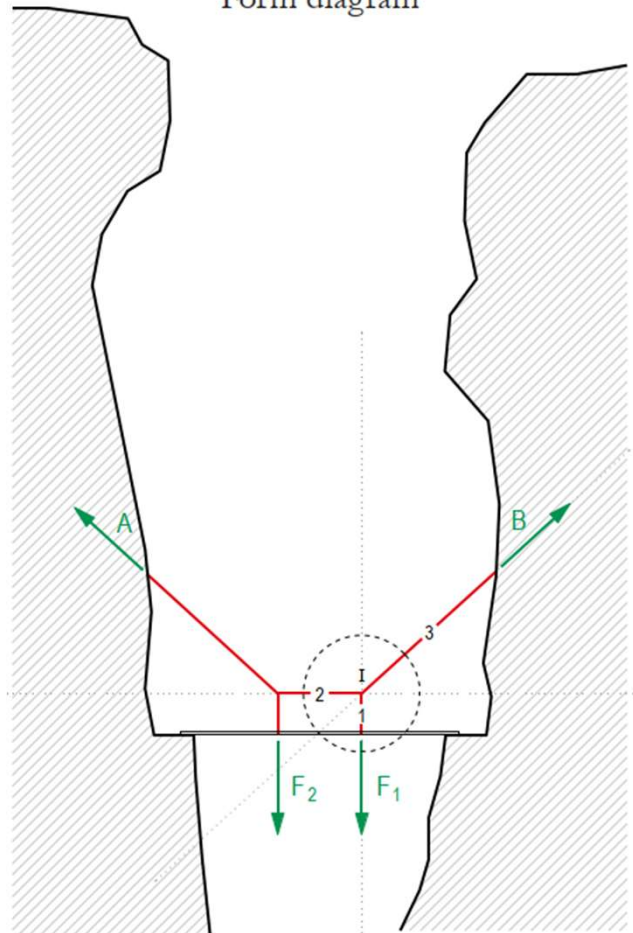
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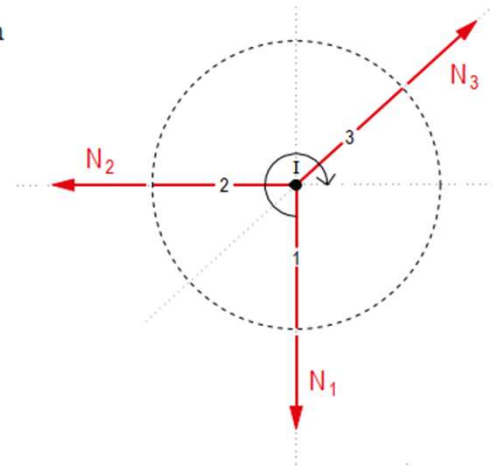
Force diagram



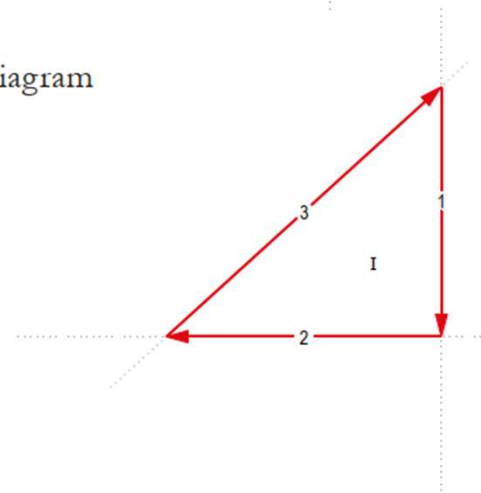
Form diagram



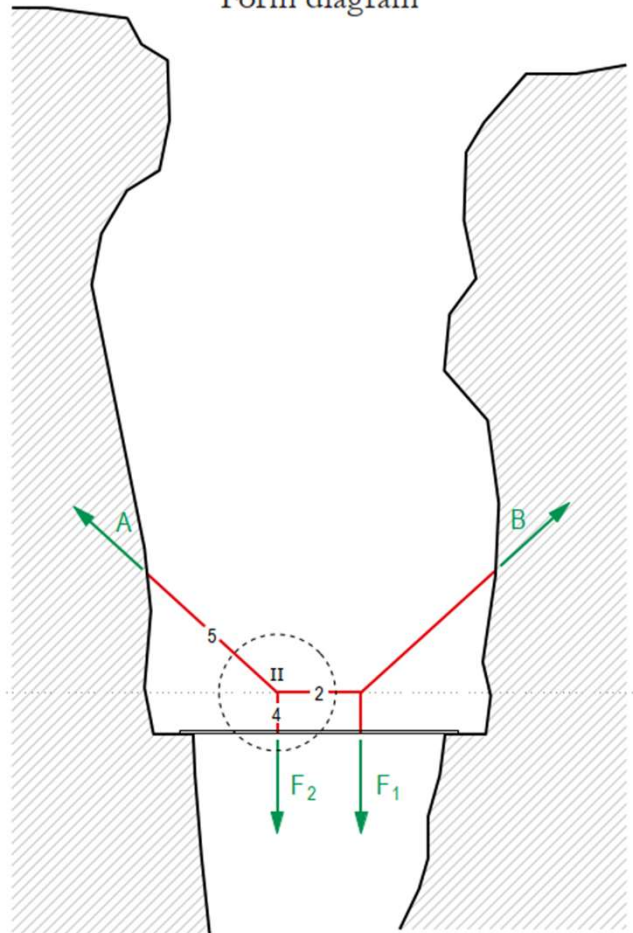
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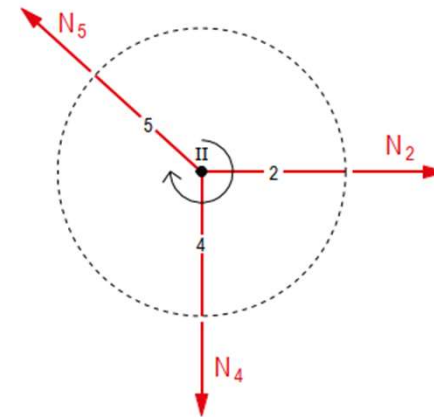
Force diagram



Form diagram



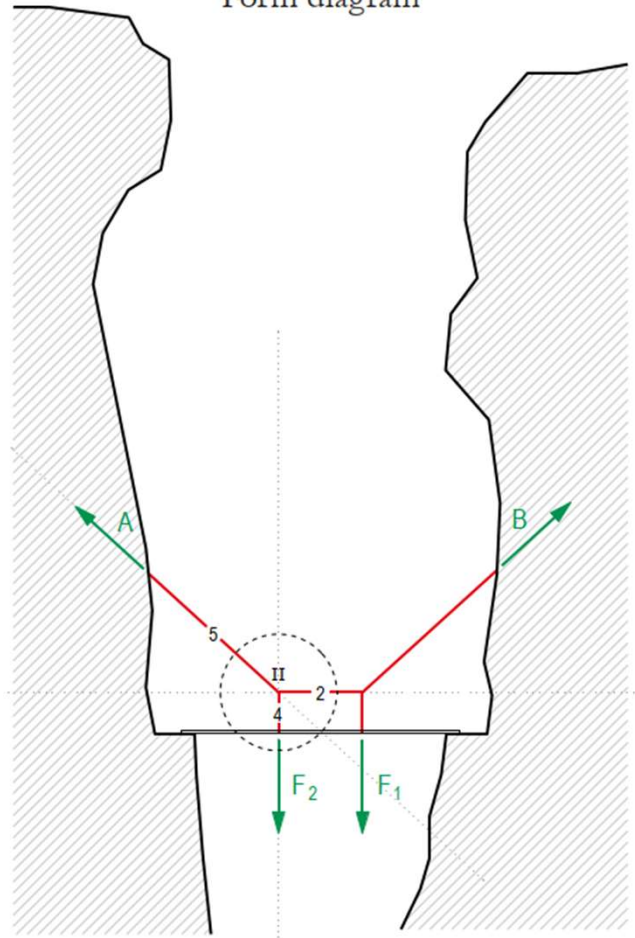
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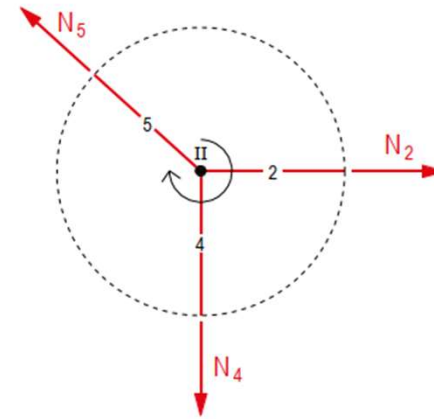
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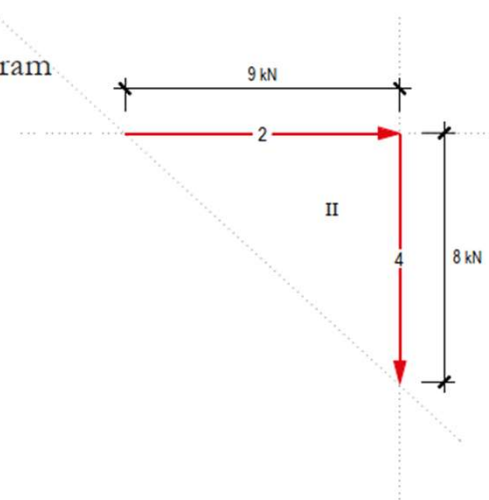
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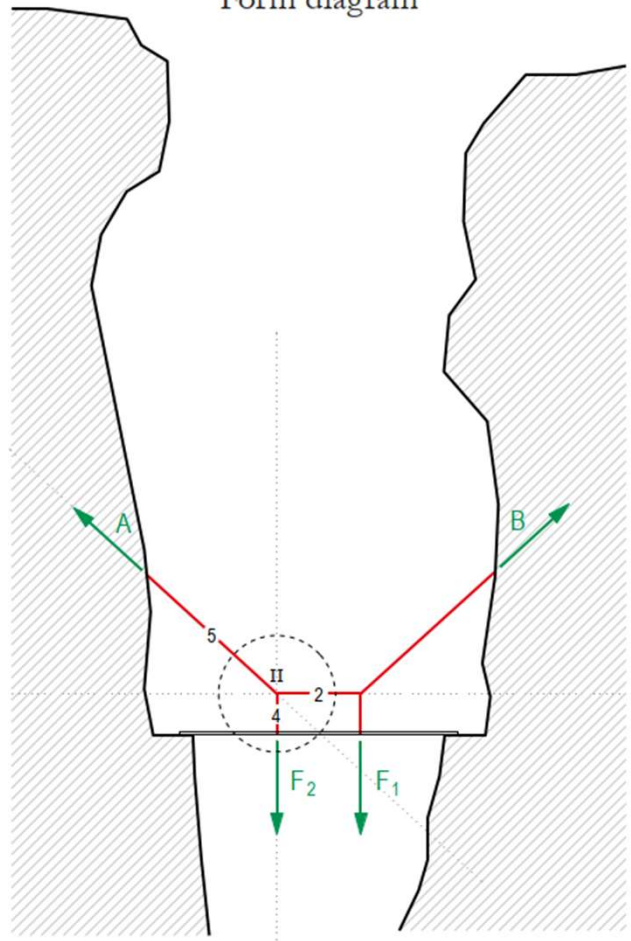
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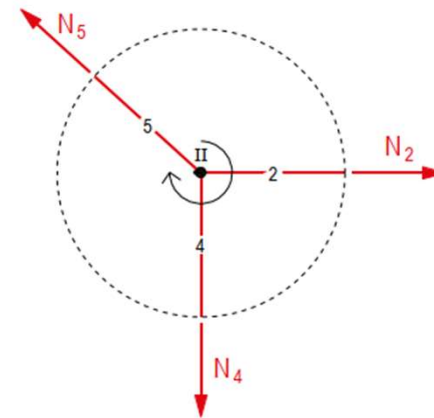
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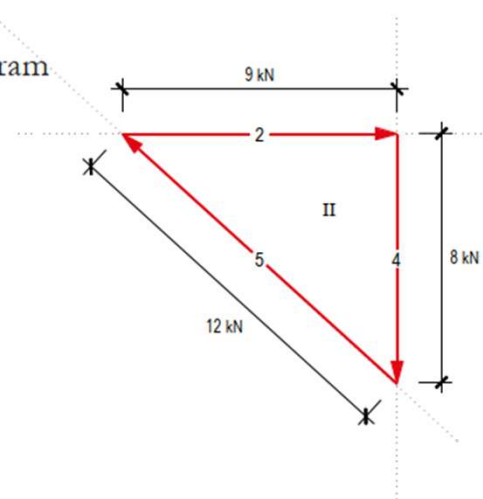
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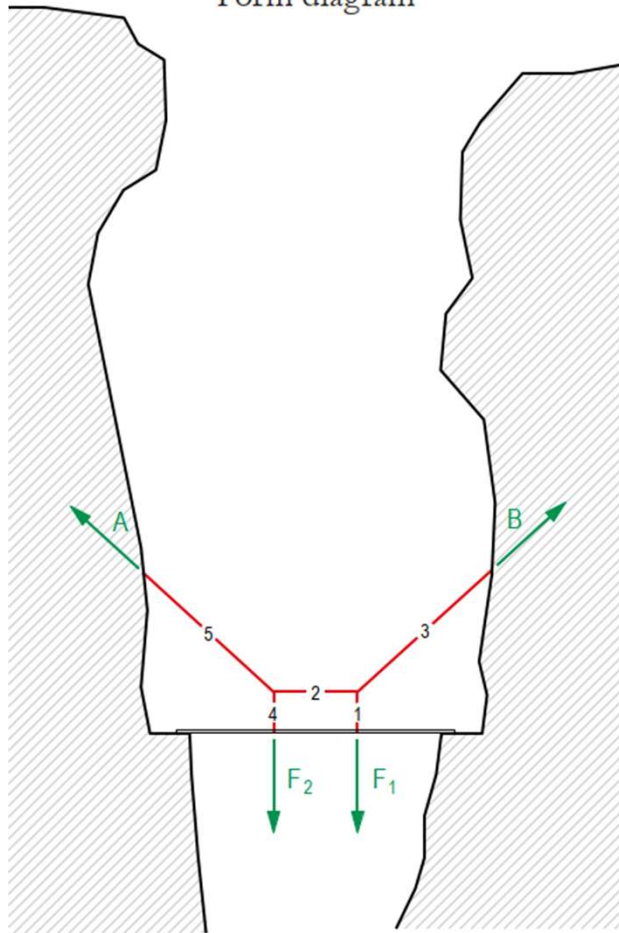
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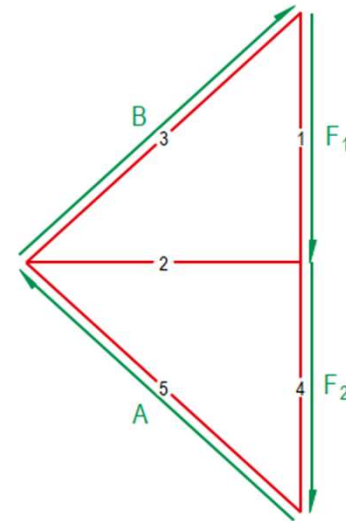
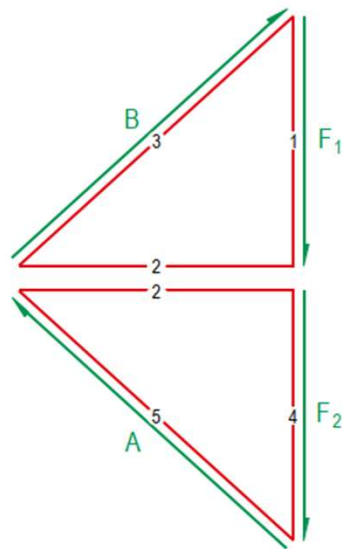
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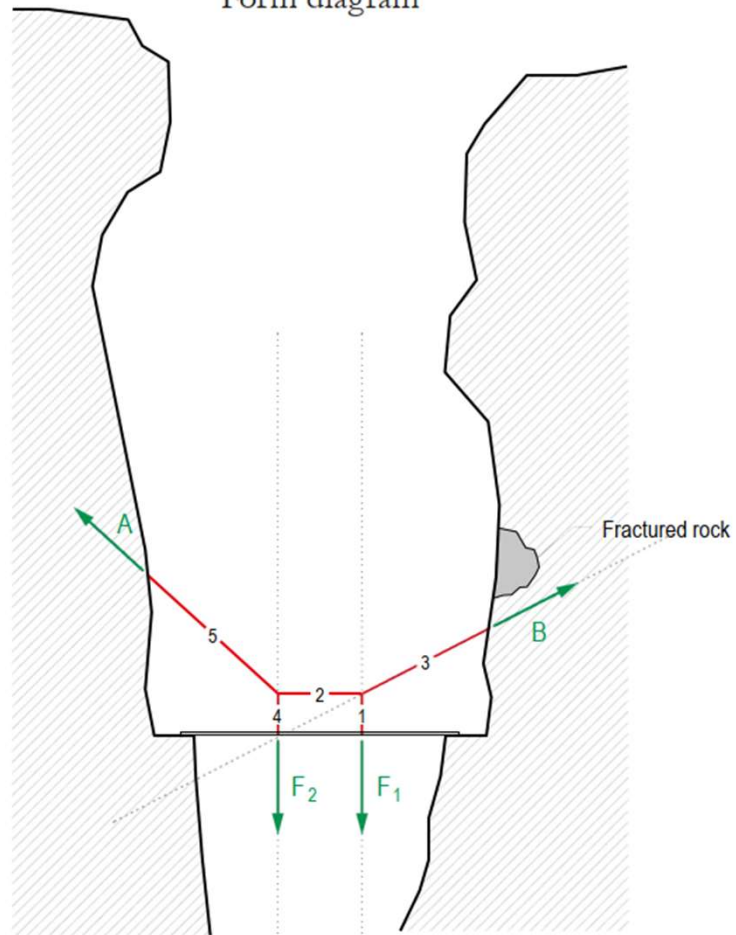
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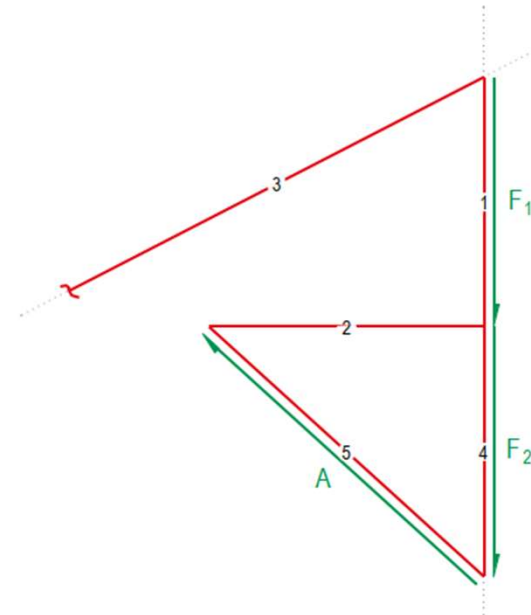
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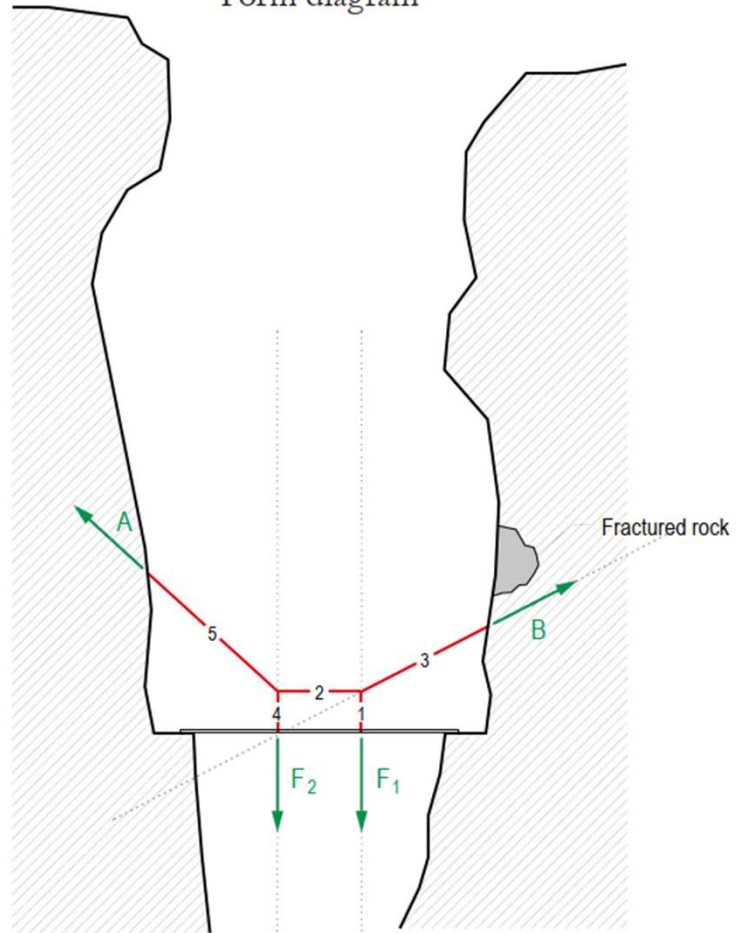
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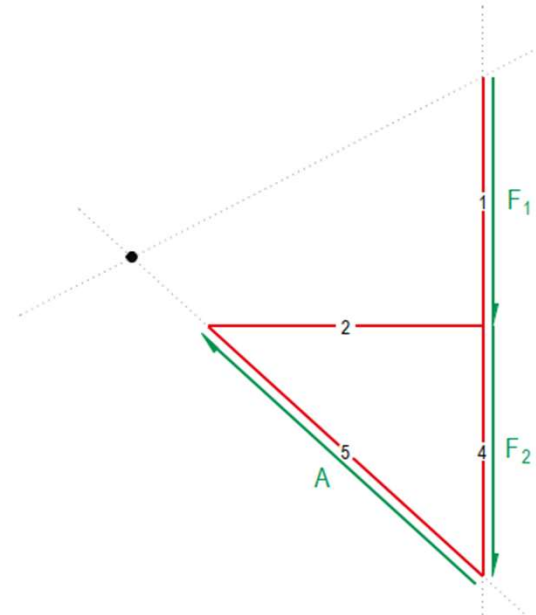
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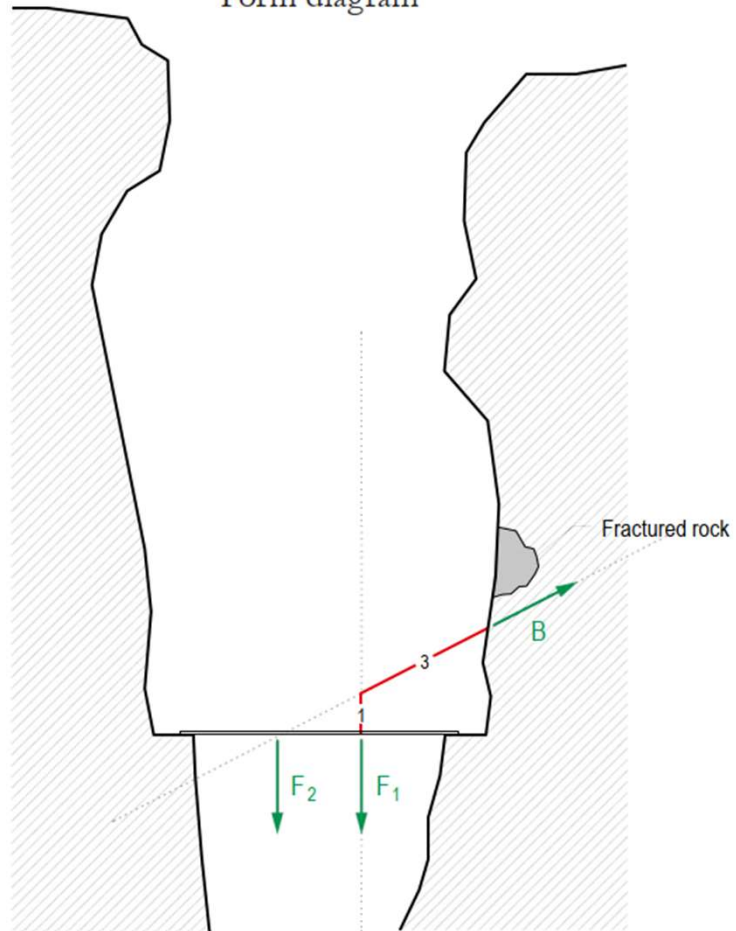
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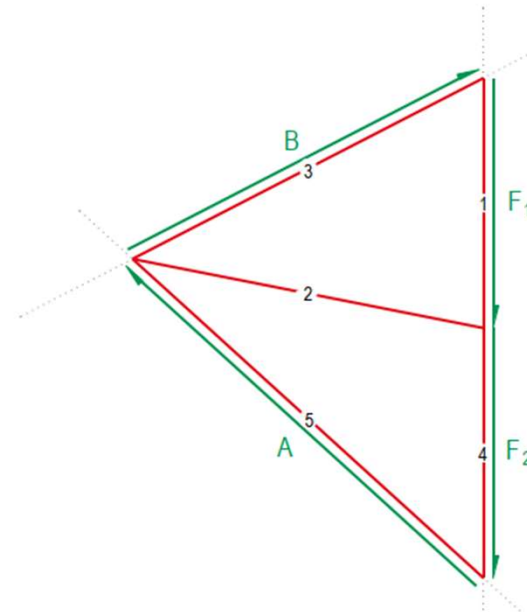
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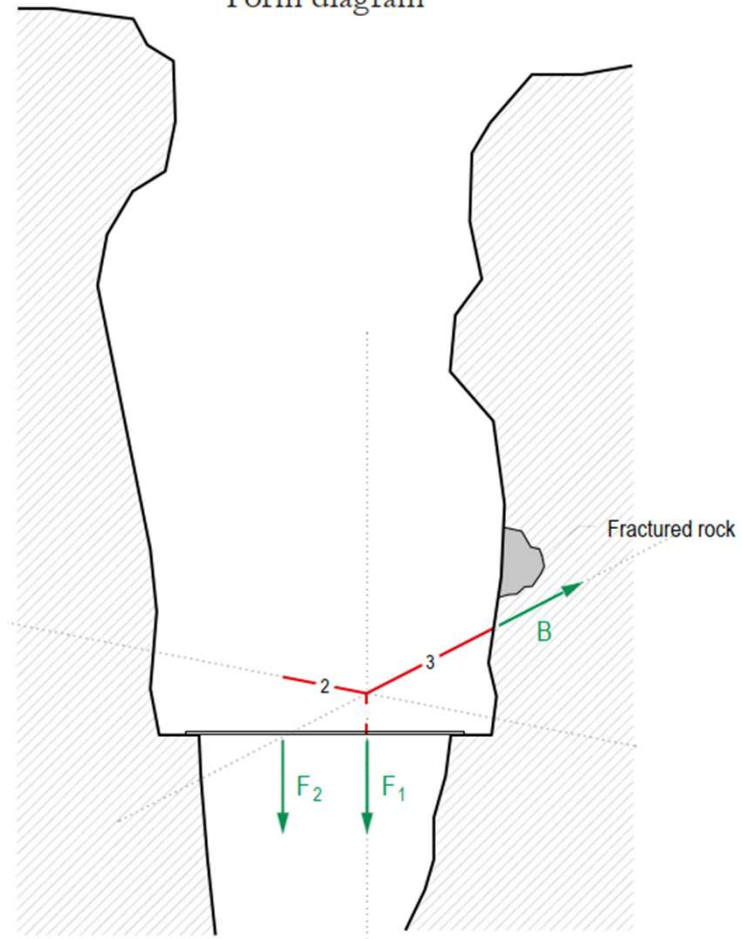
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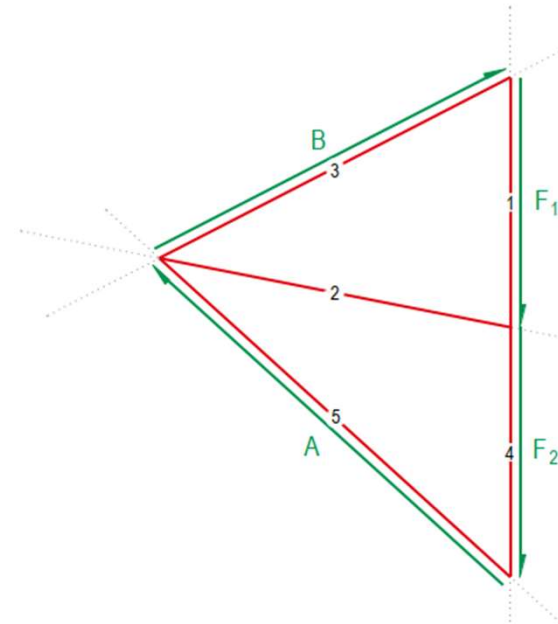
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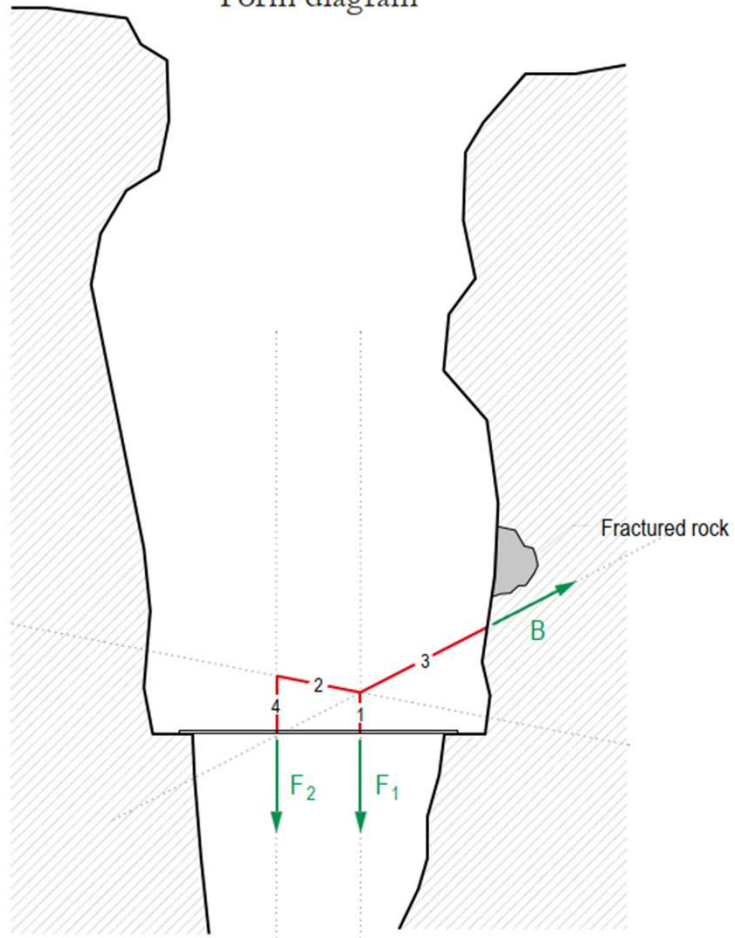
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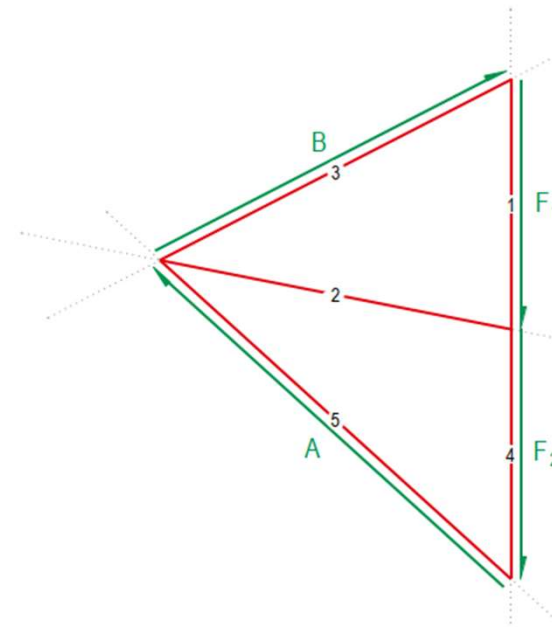
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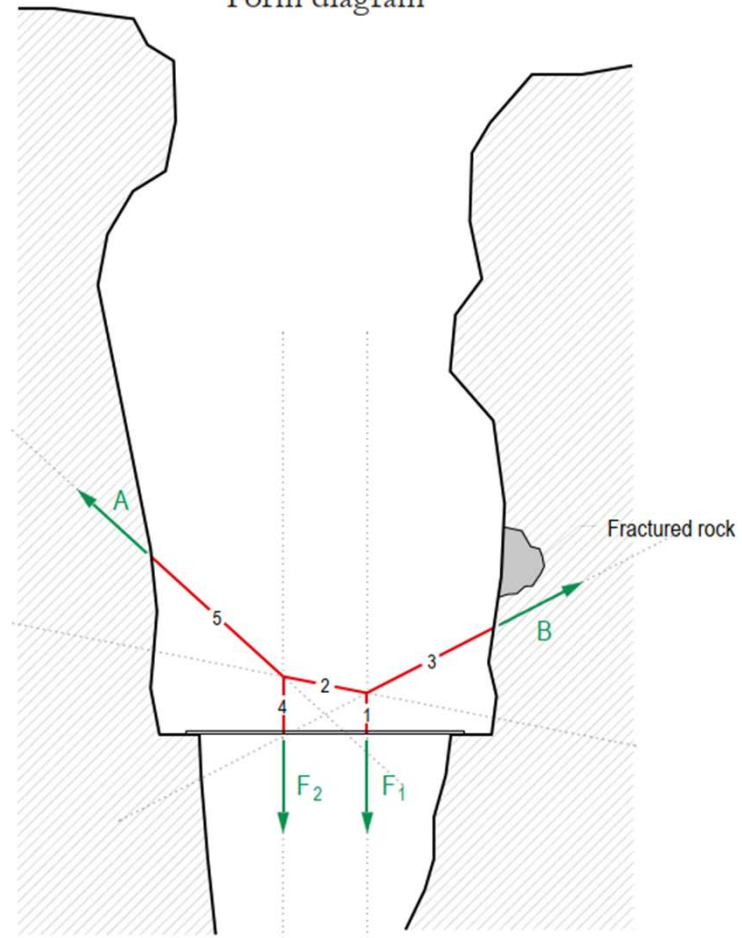
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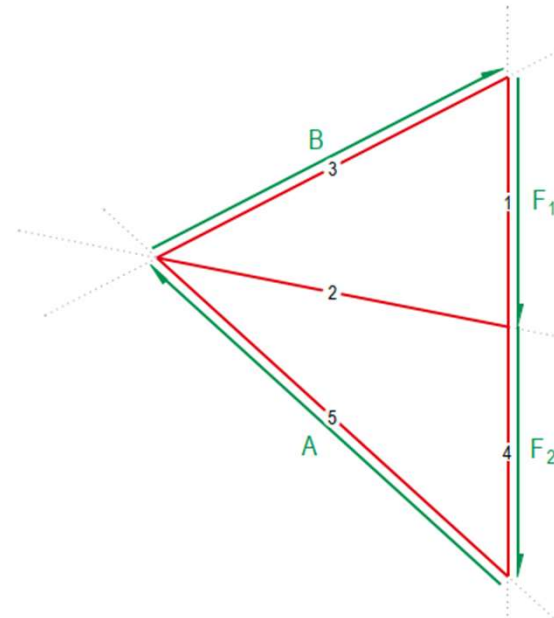
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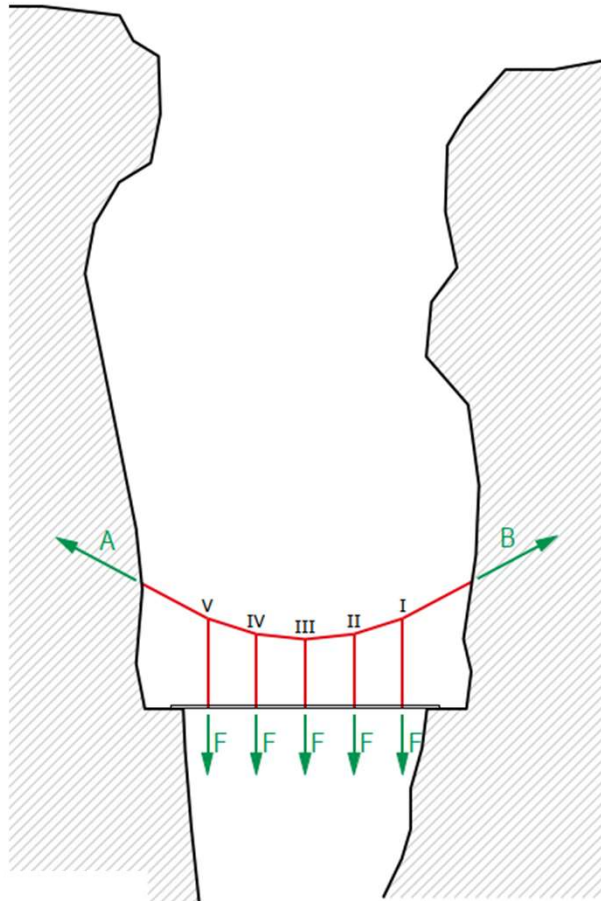
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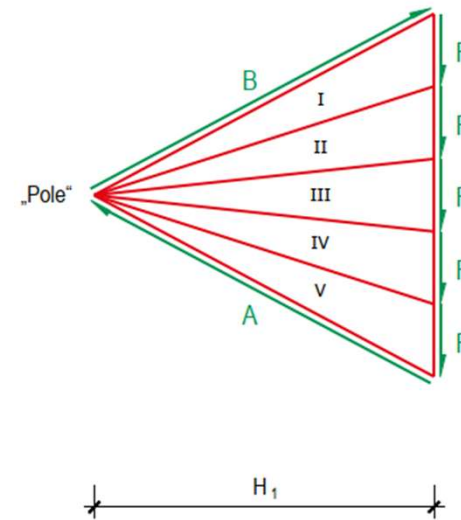
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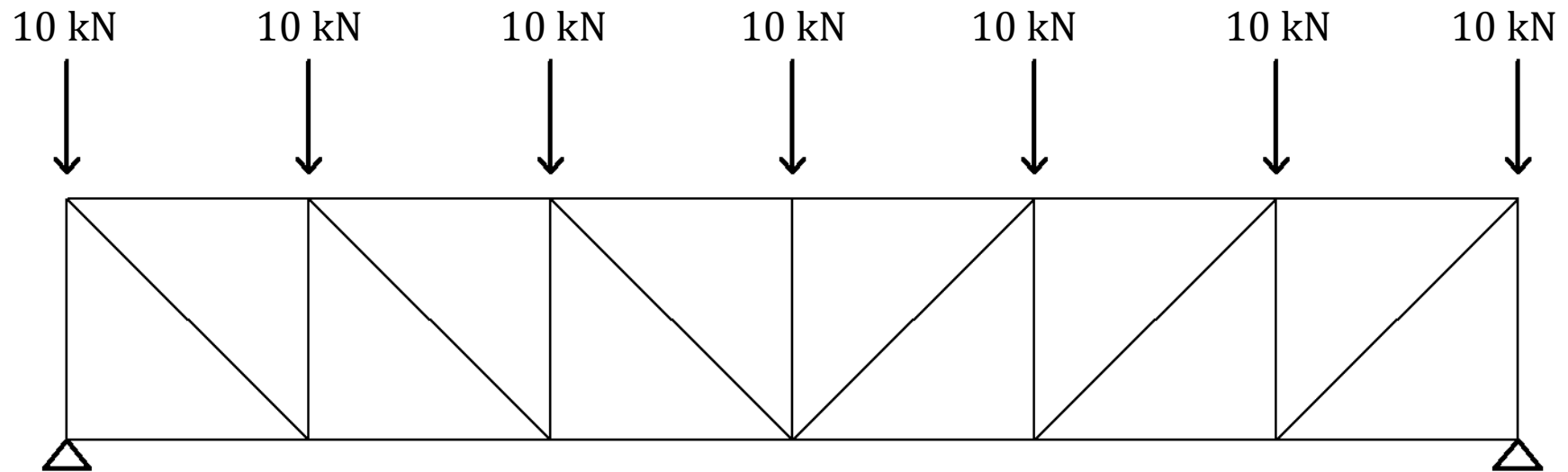
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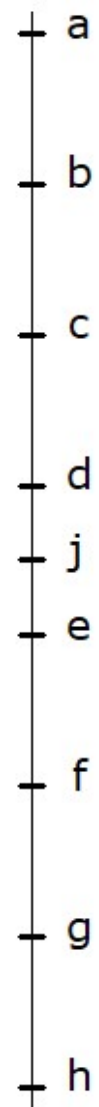
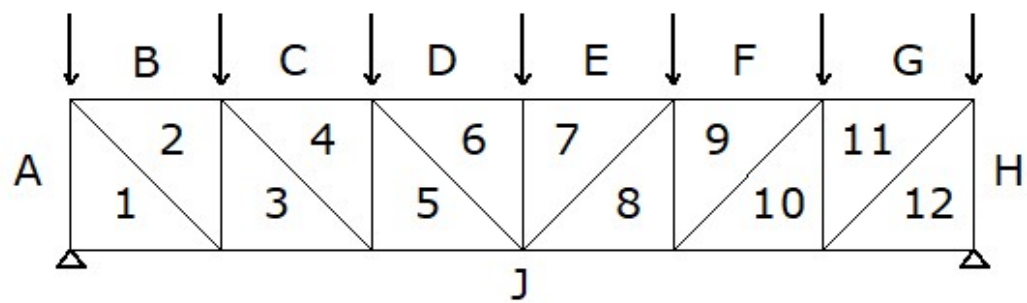


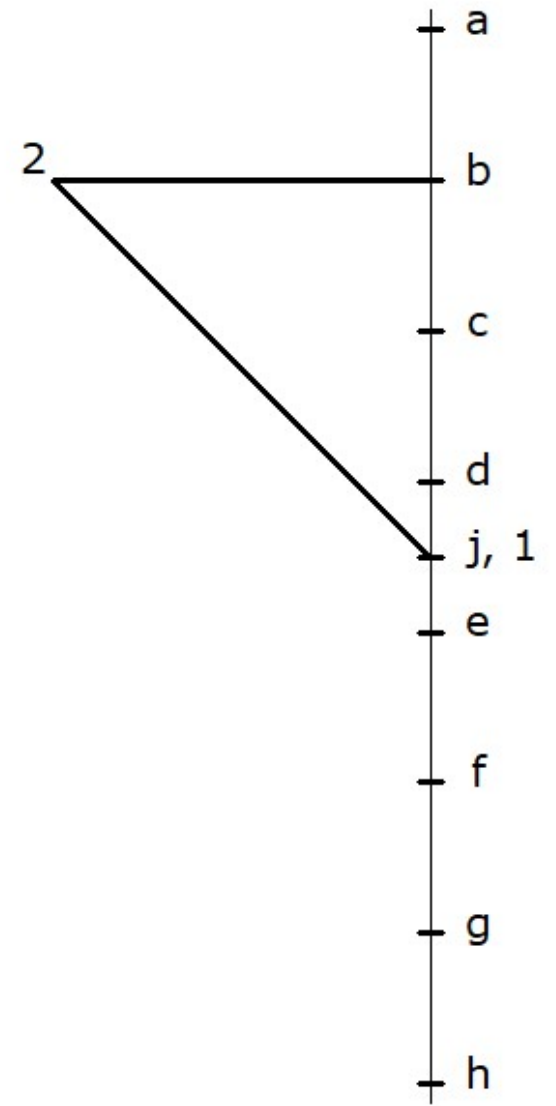
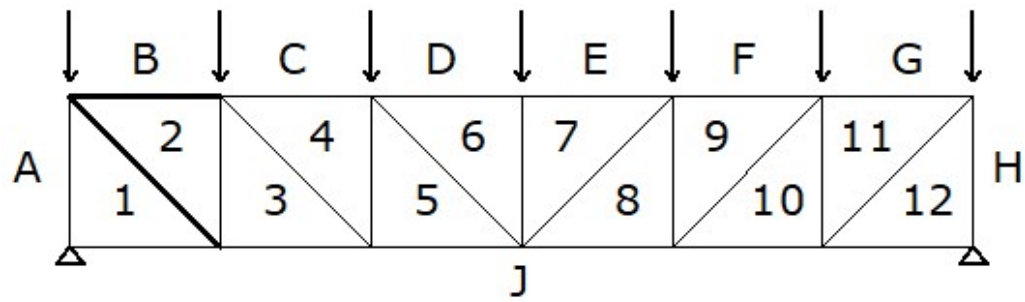
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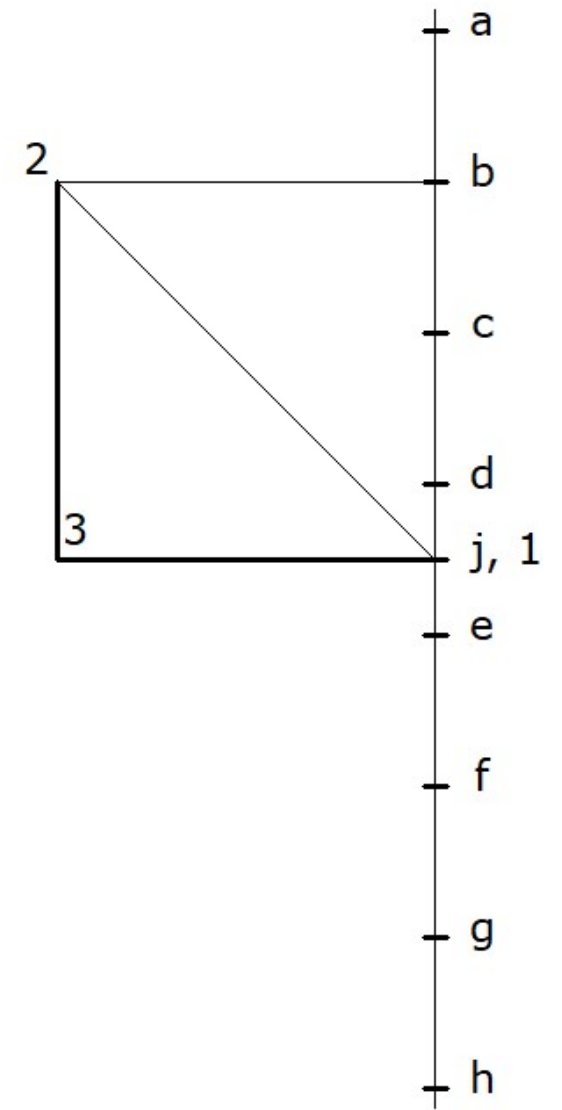
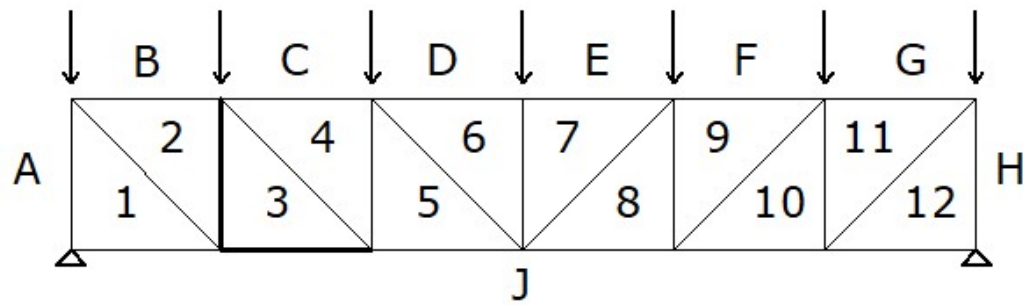


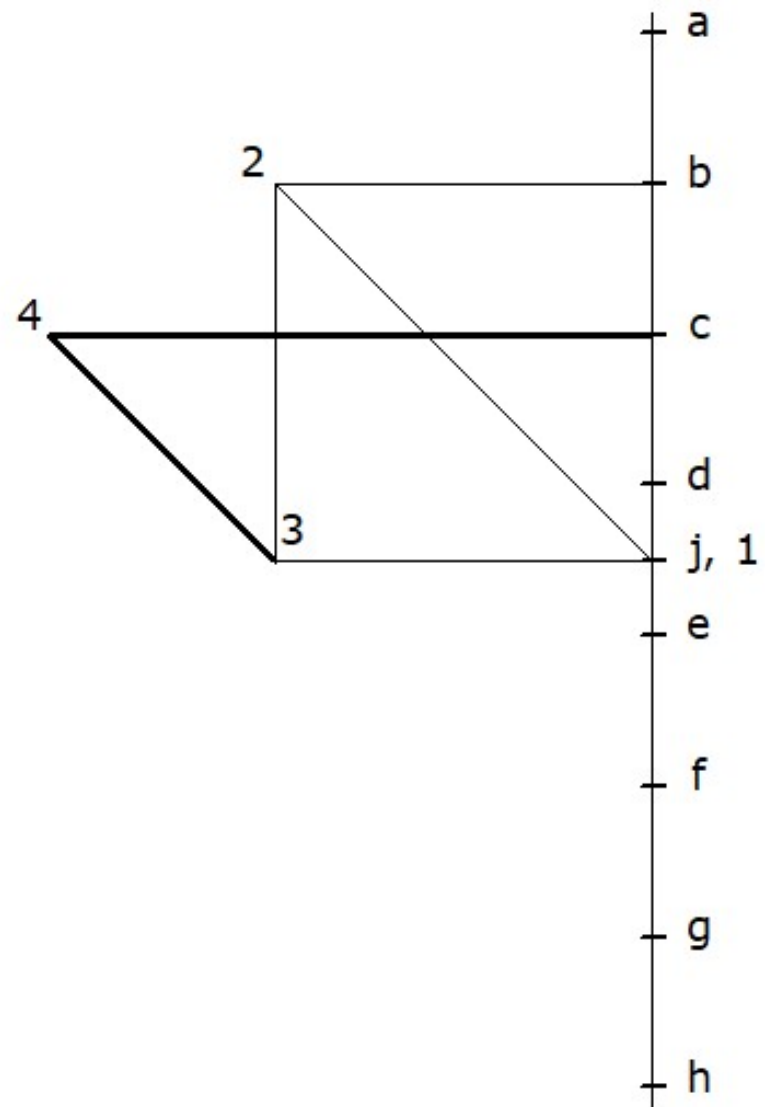
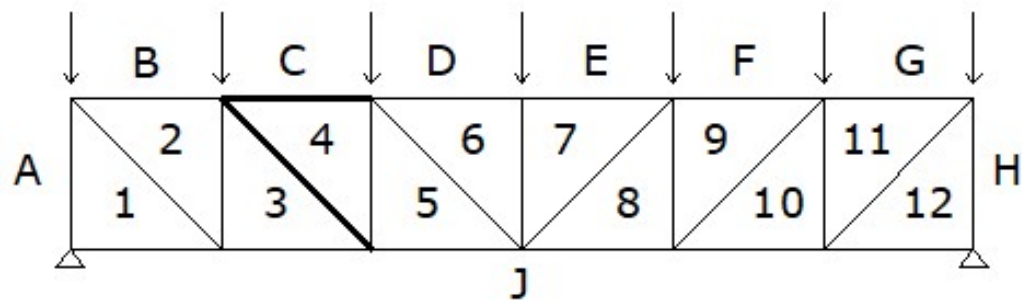
Example

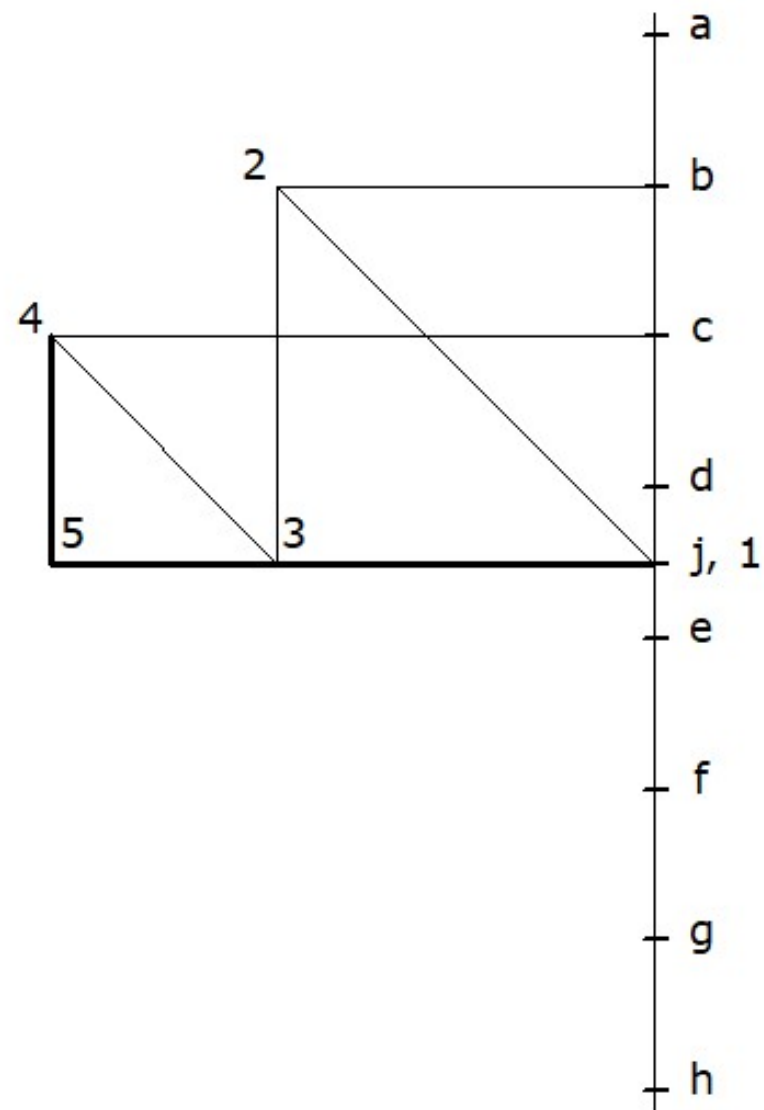
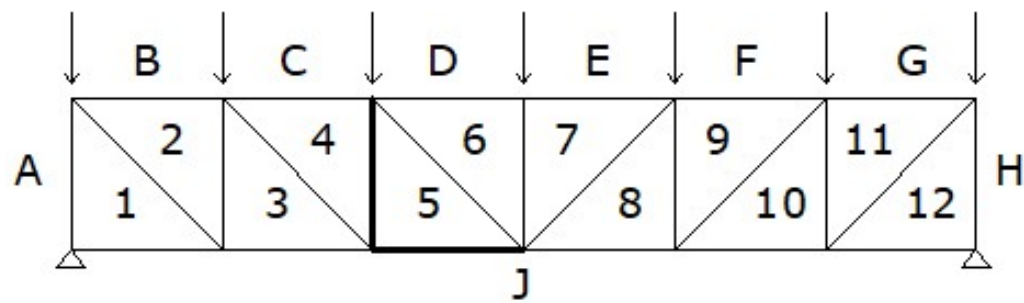


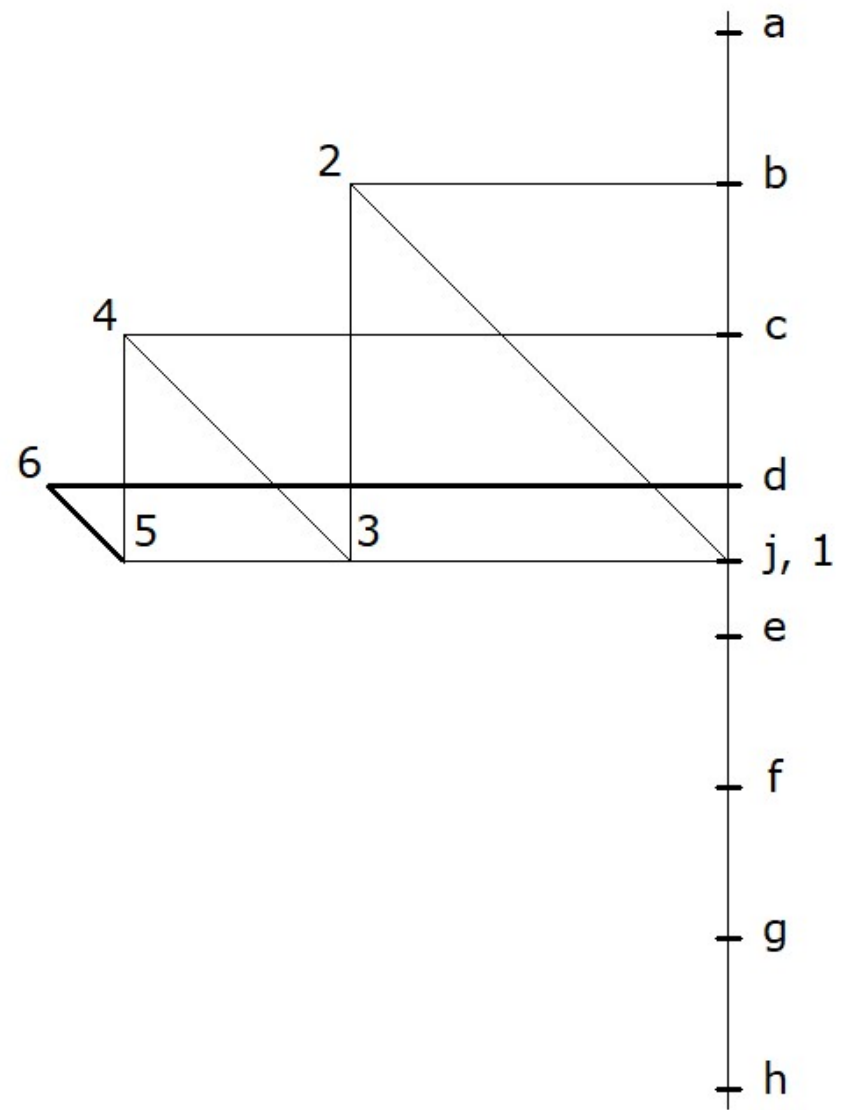
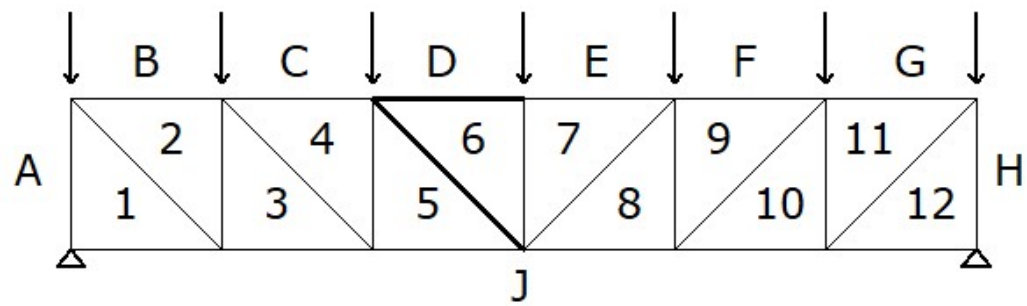


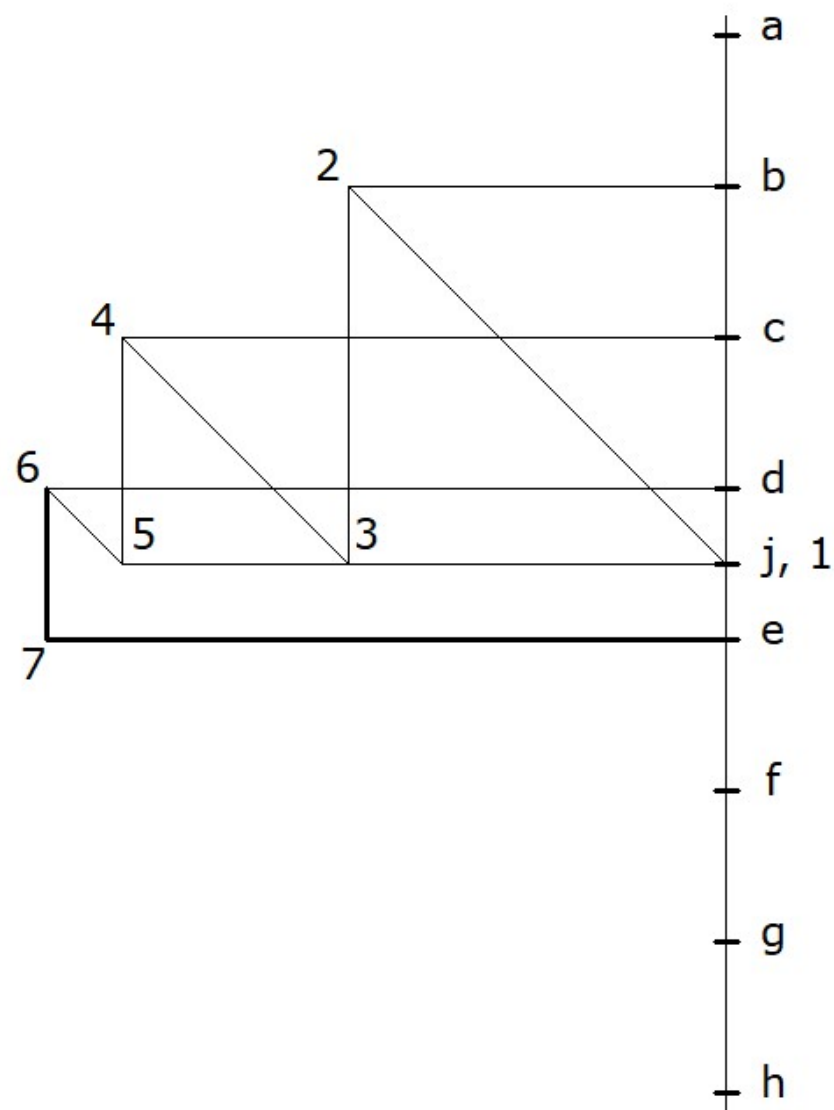
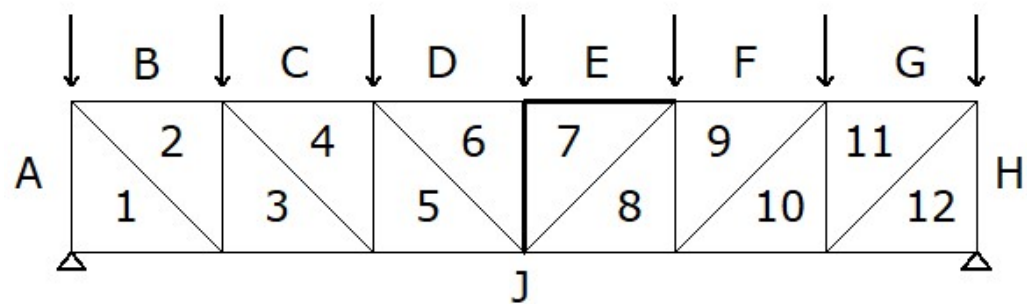


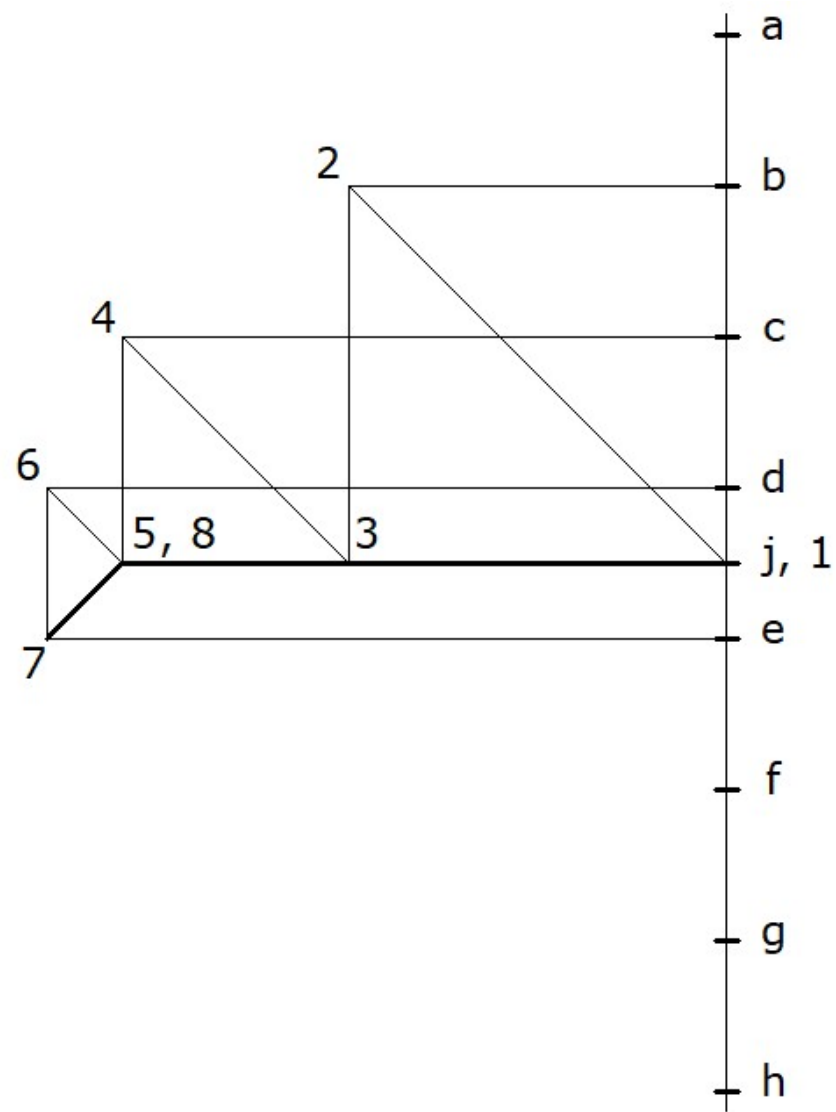
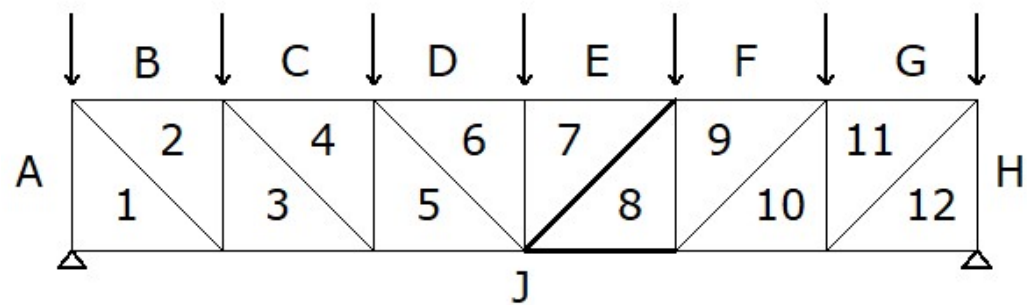


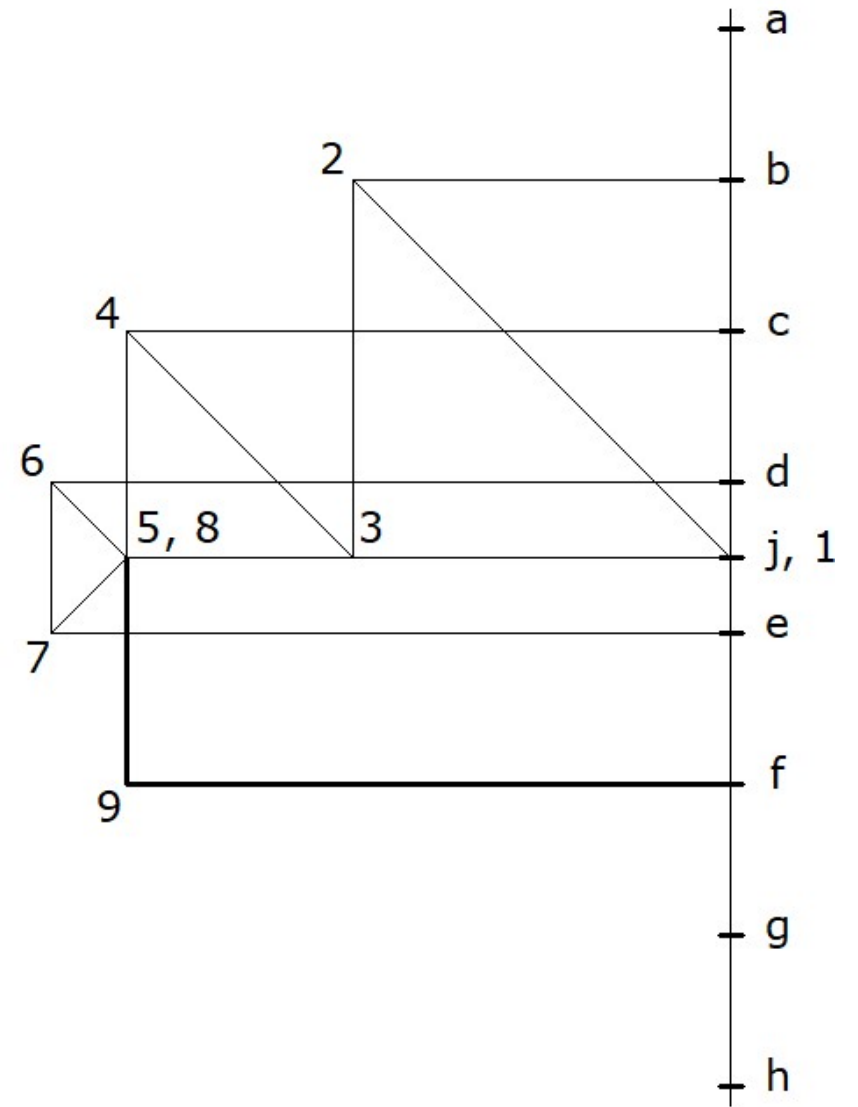
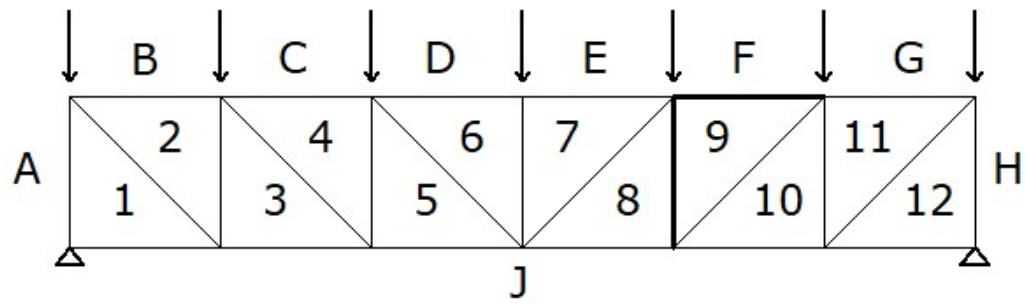


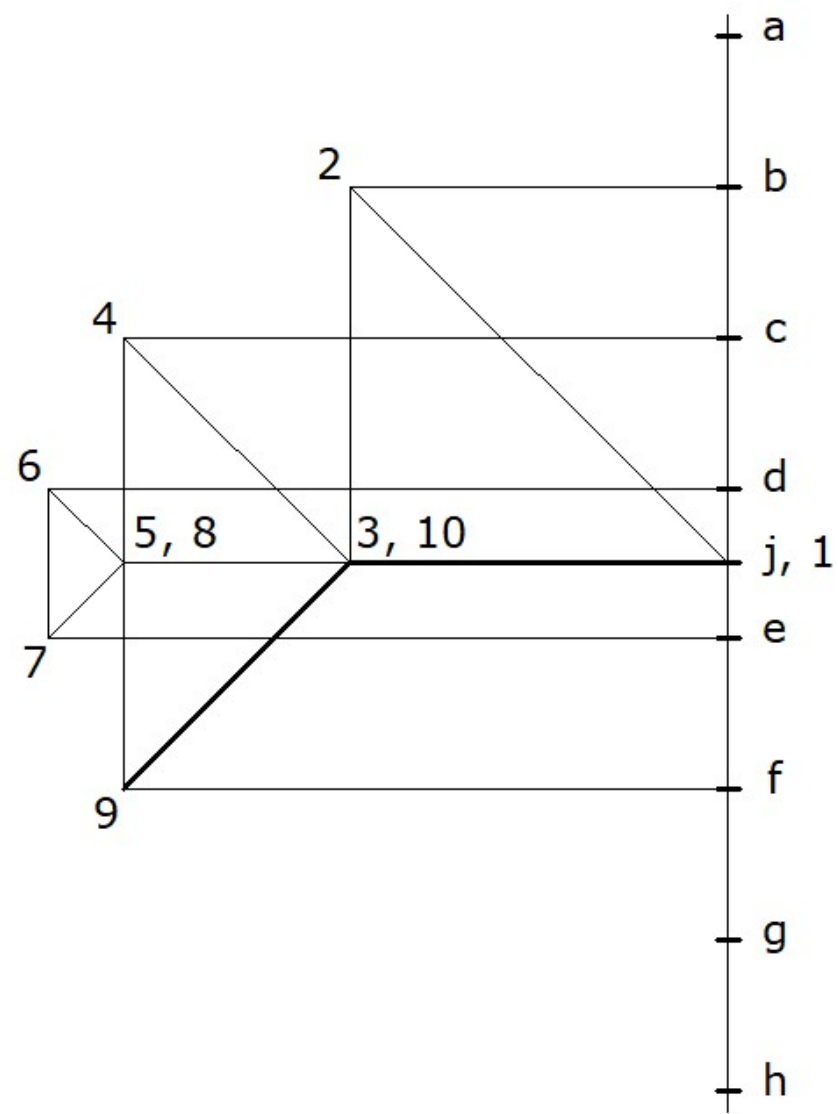
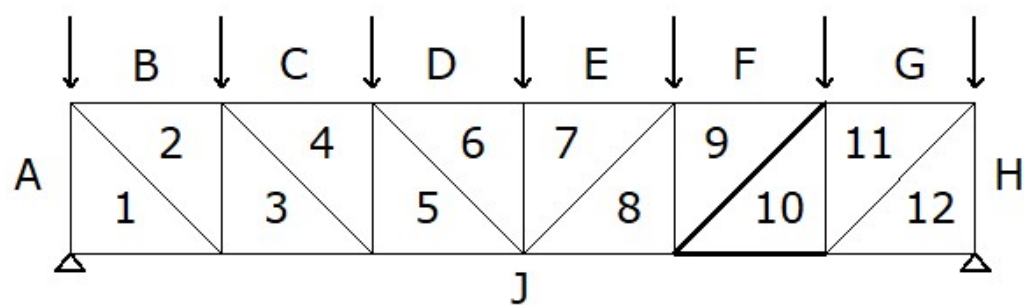


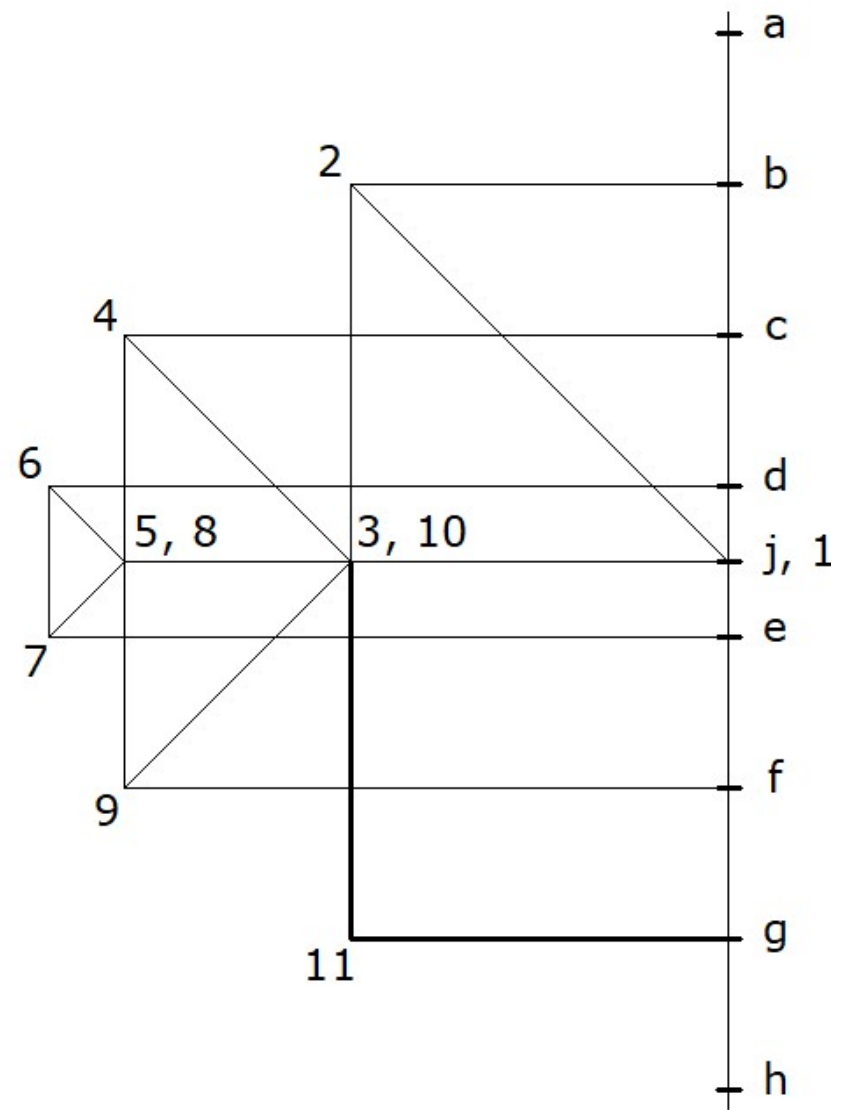
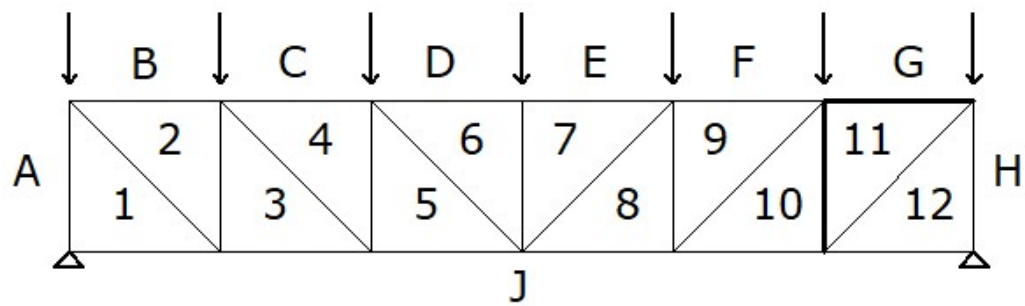


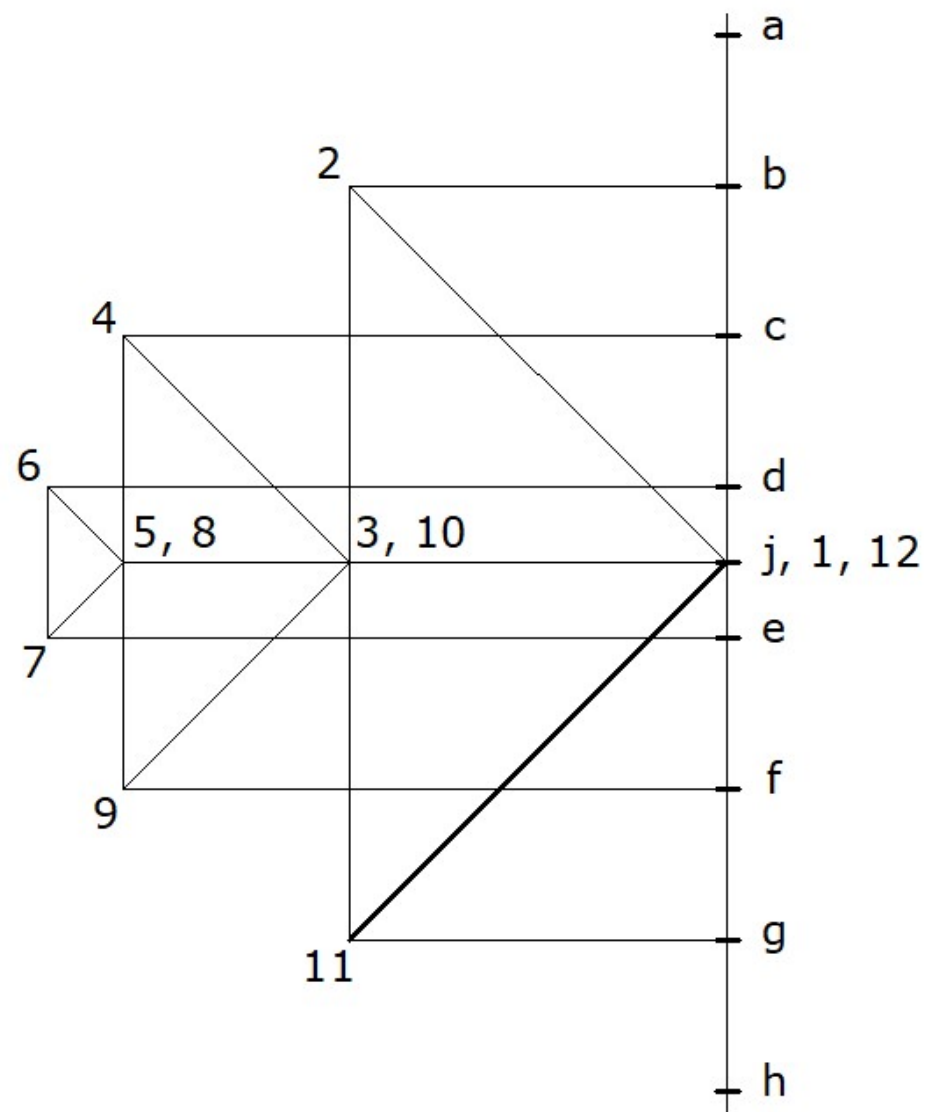
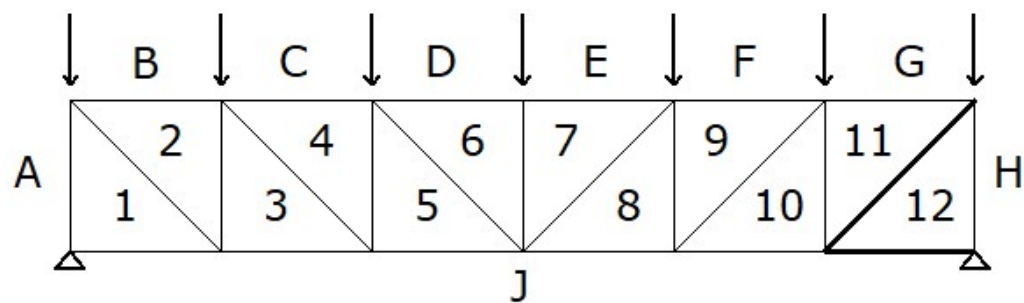


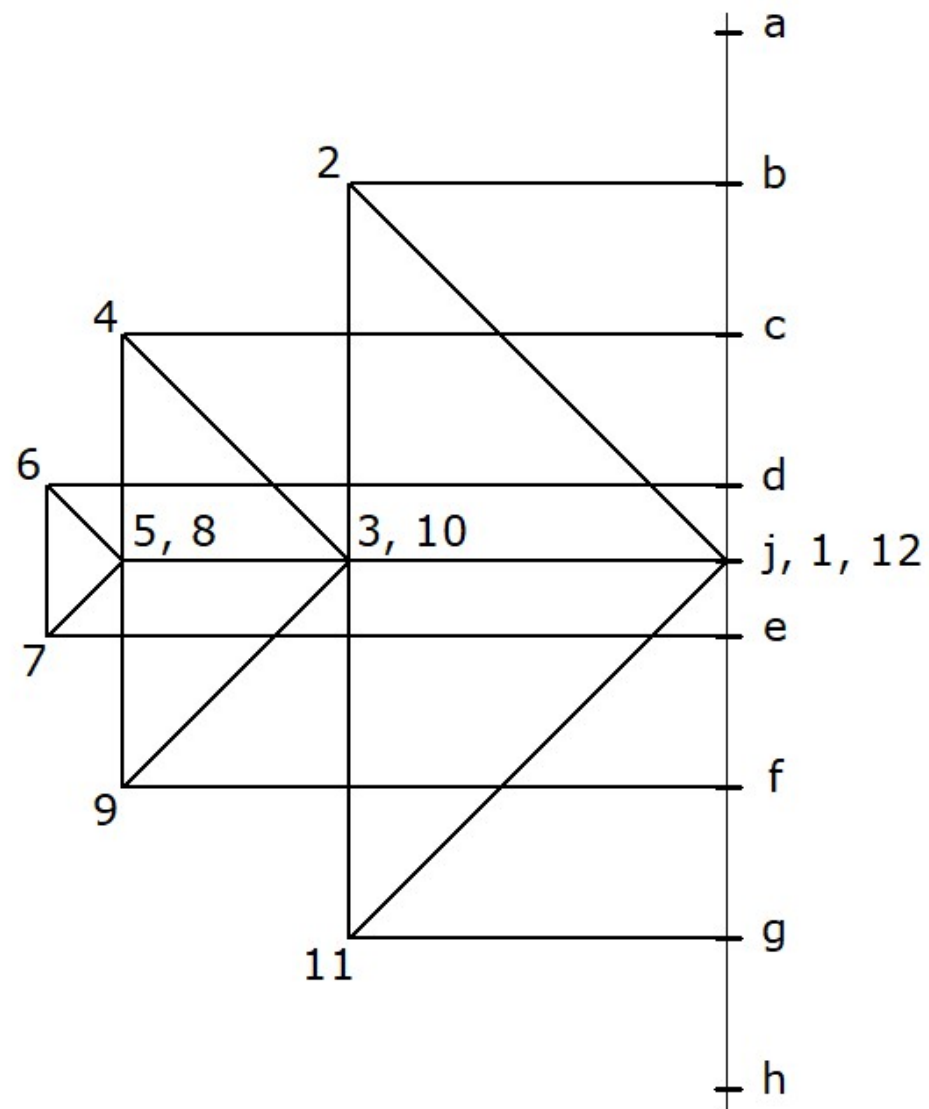
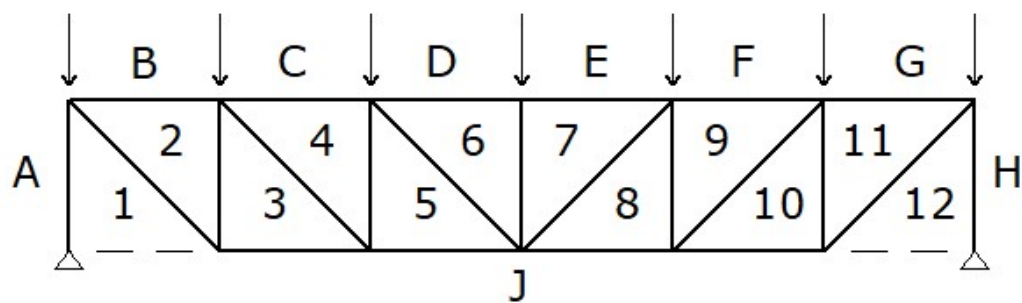




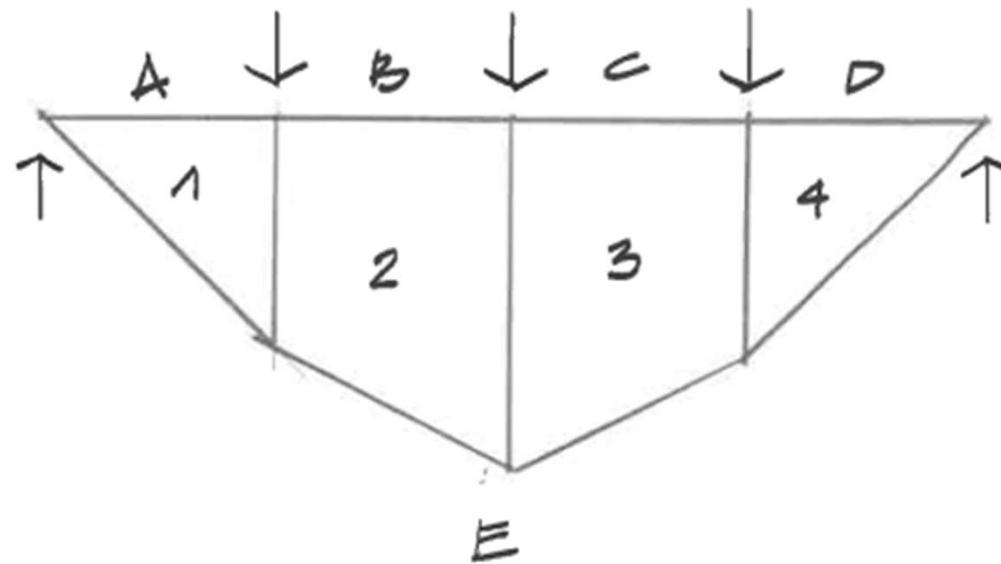




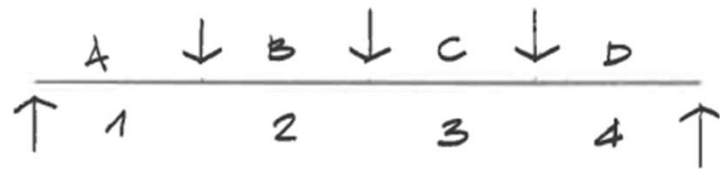




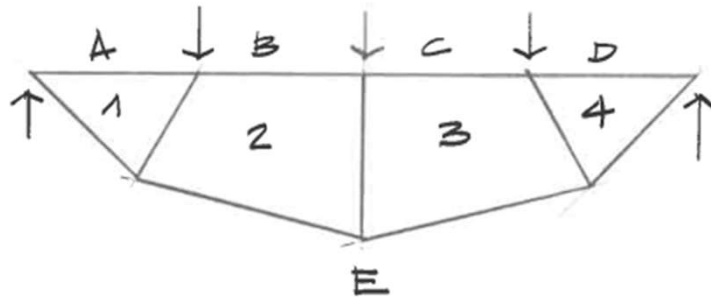
Example



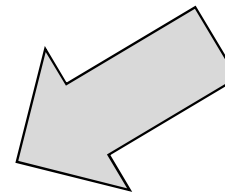
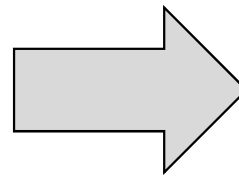
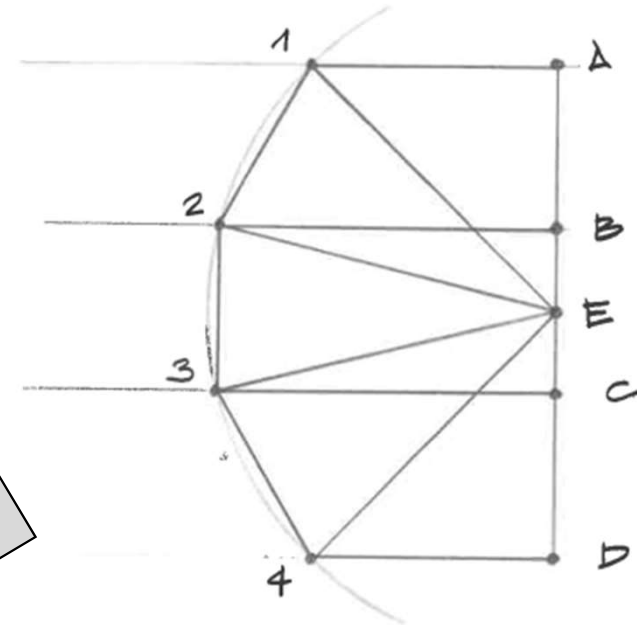
Form diagram



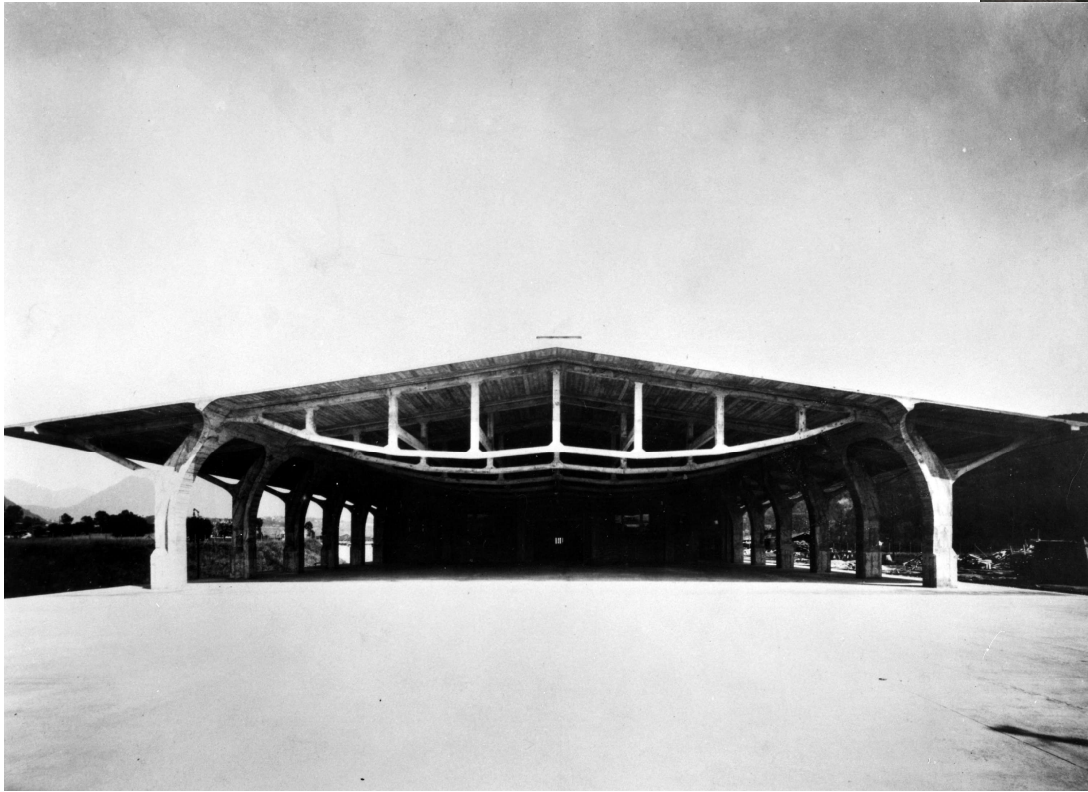
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Force polygon

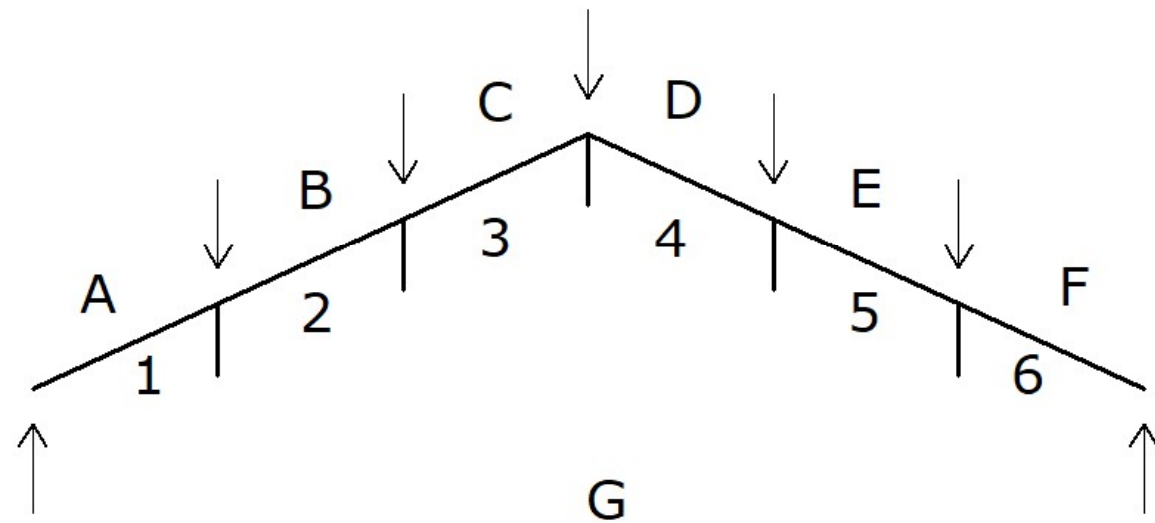


Example

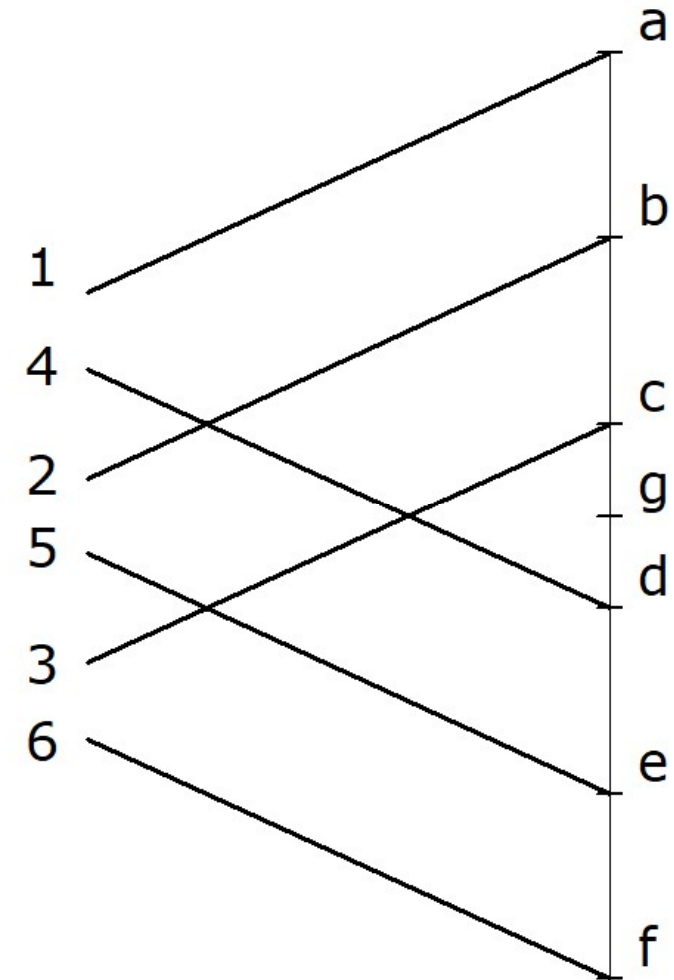


Magazzini Generali warehouse
Chiasso, Switzerland
Robert Maillart (1924)

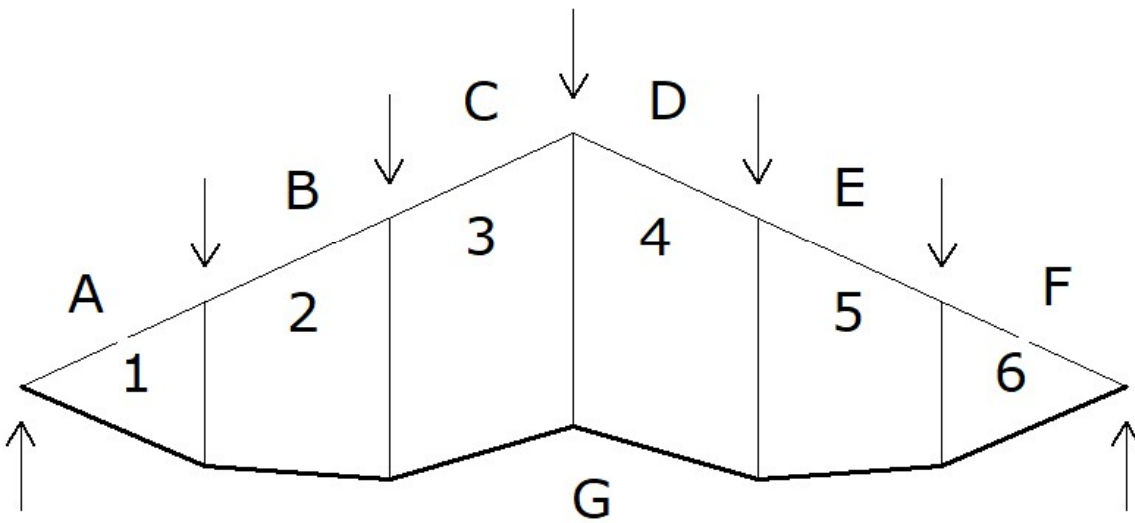
Form diagram



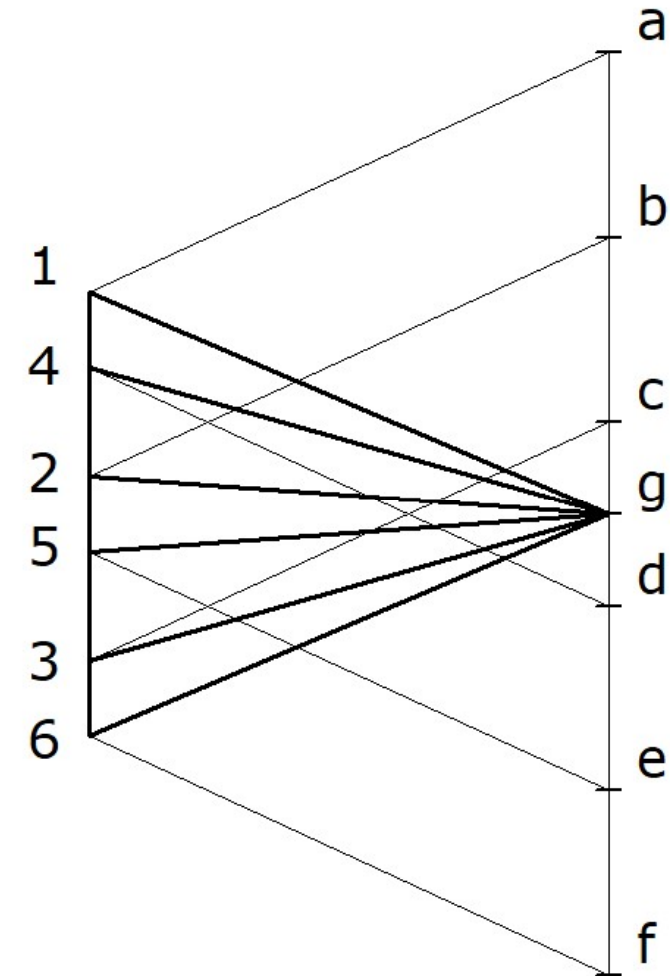
Force polygon



Form diagram



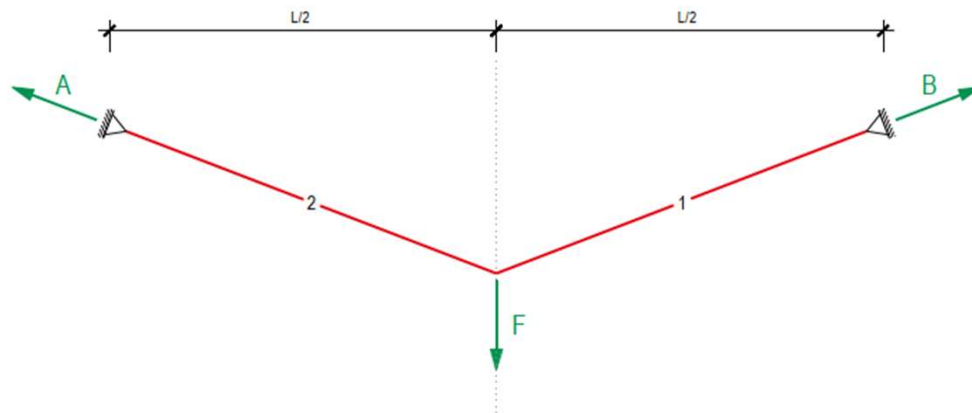
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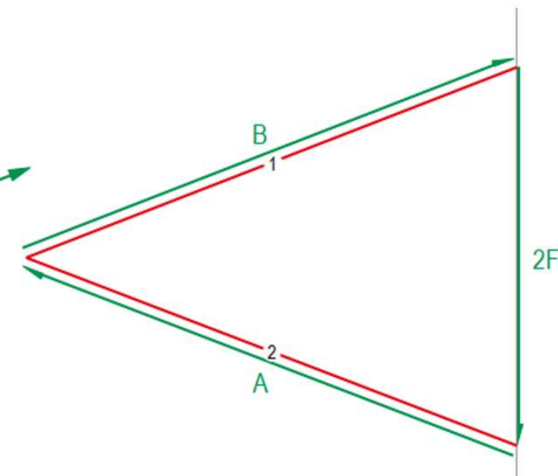
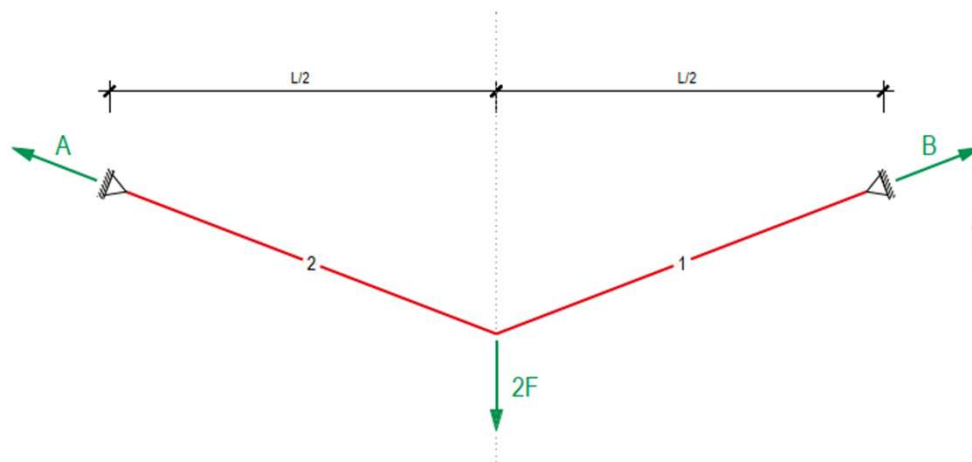
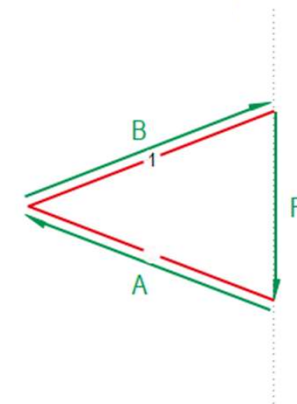
Relationship between form and force



Form diagram

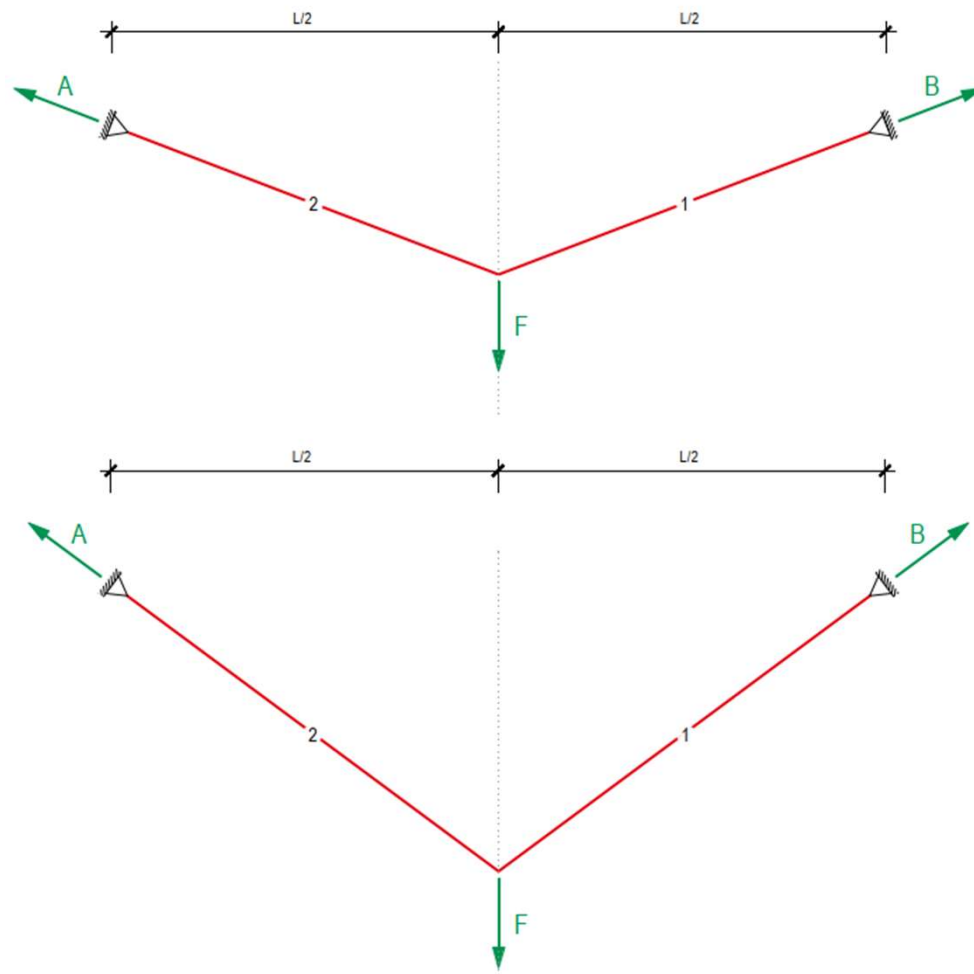


Force diagram





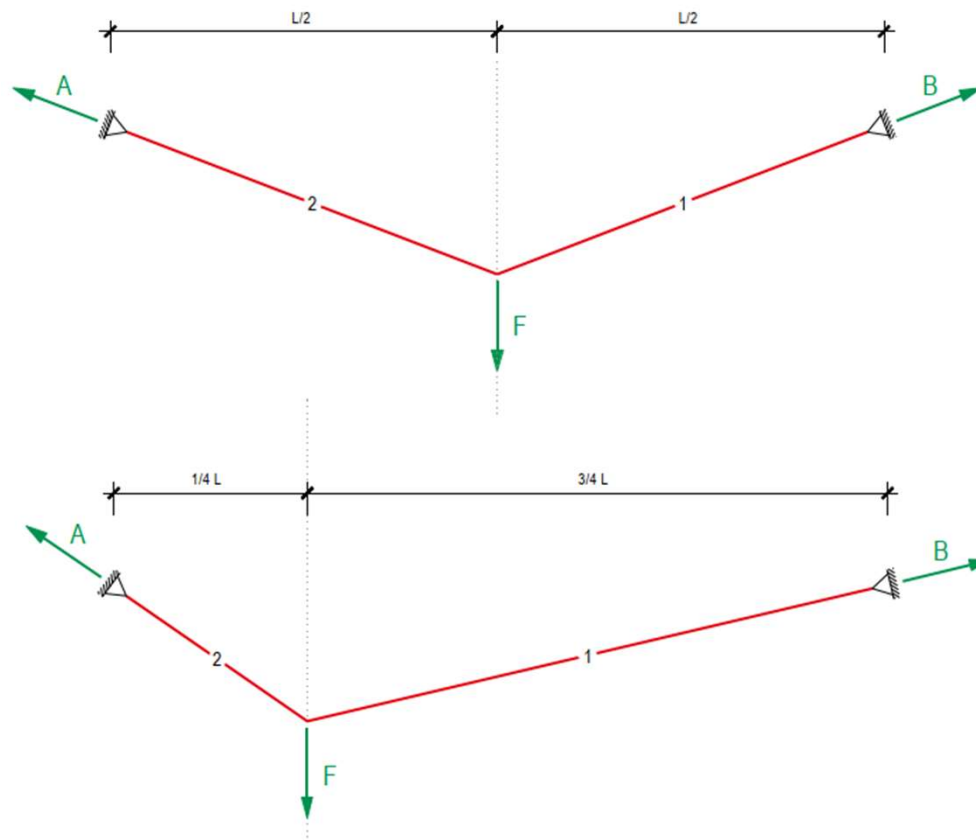
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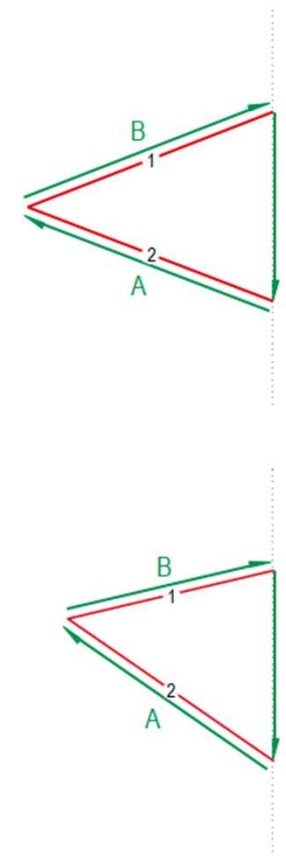
Force diagram



Form diagram

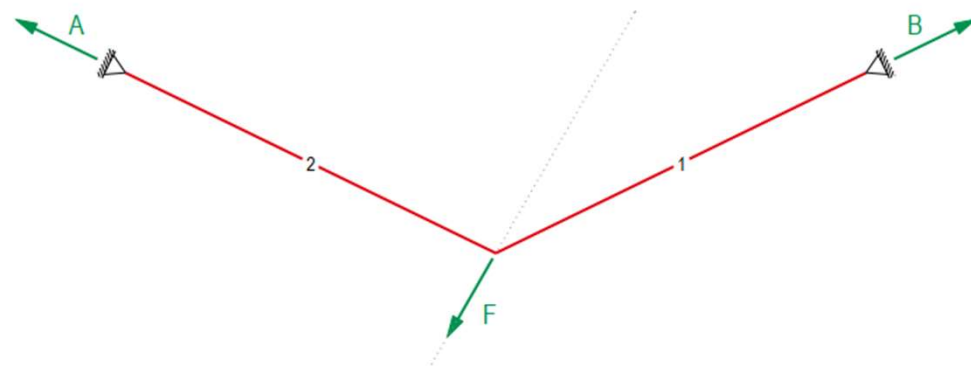
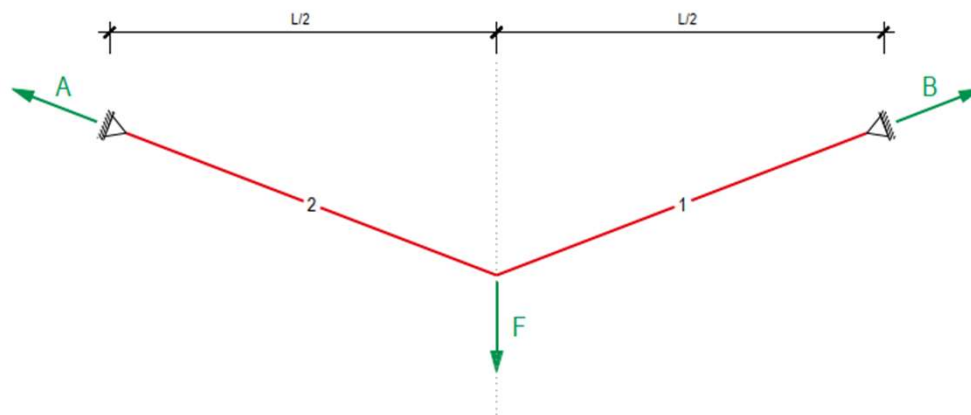


Force diagram

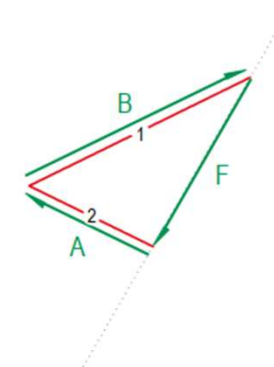
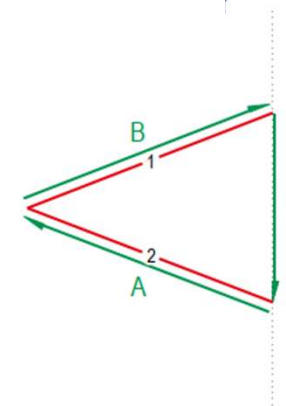




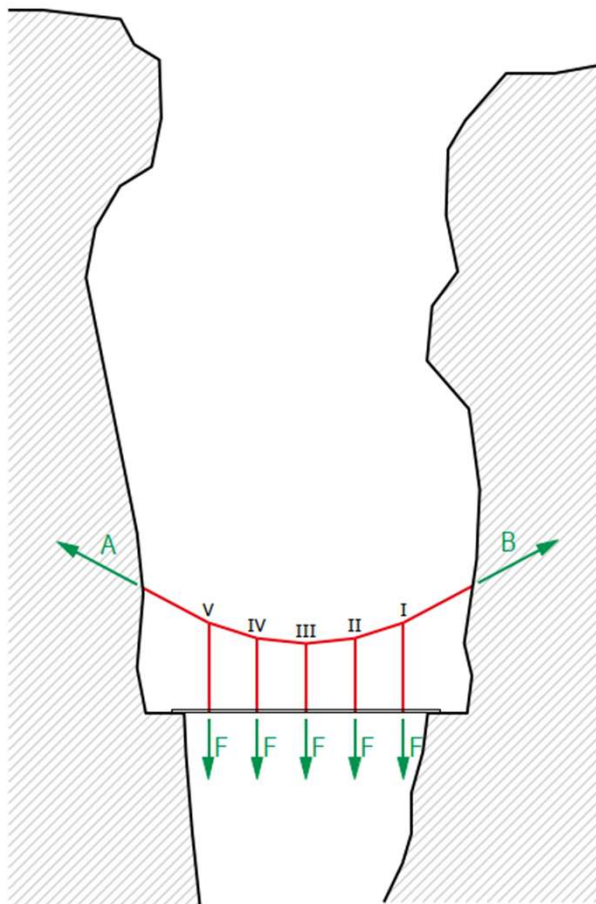
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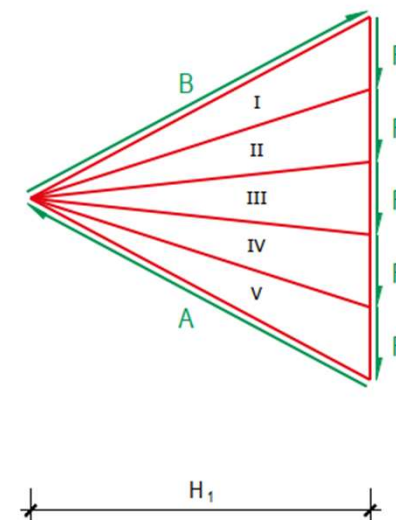
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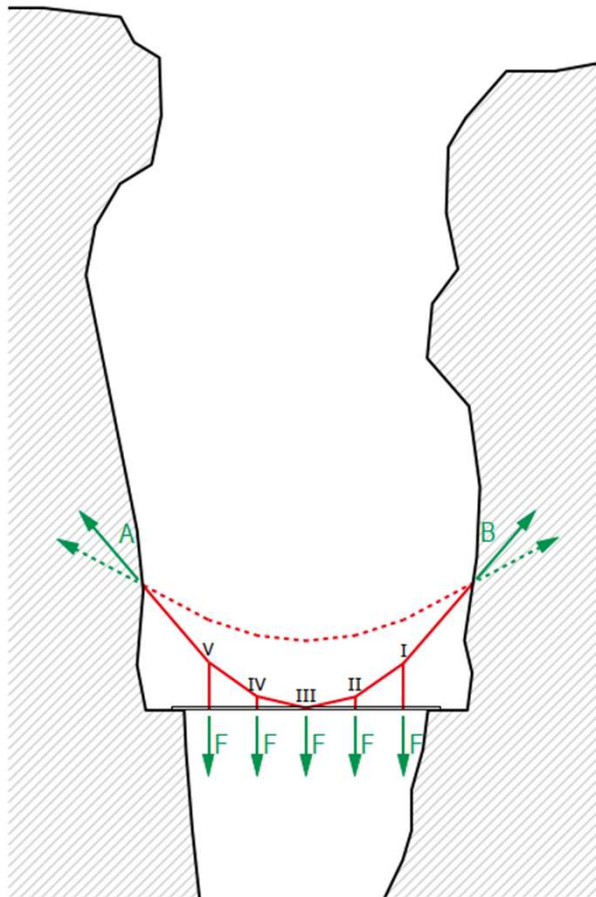
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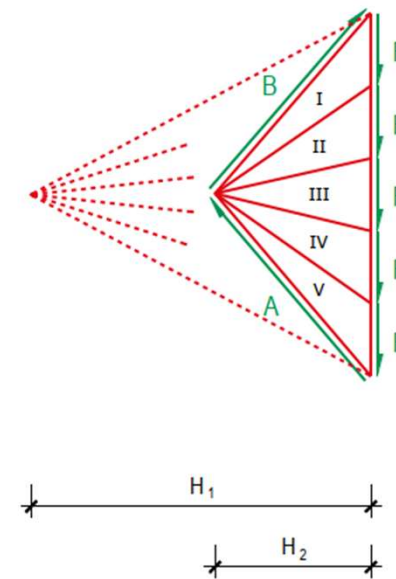
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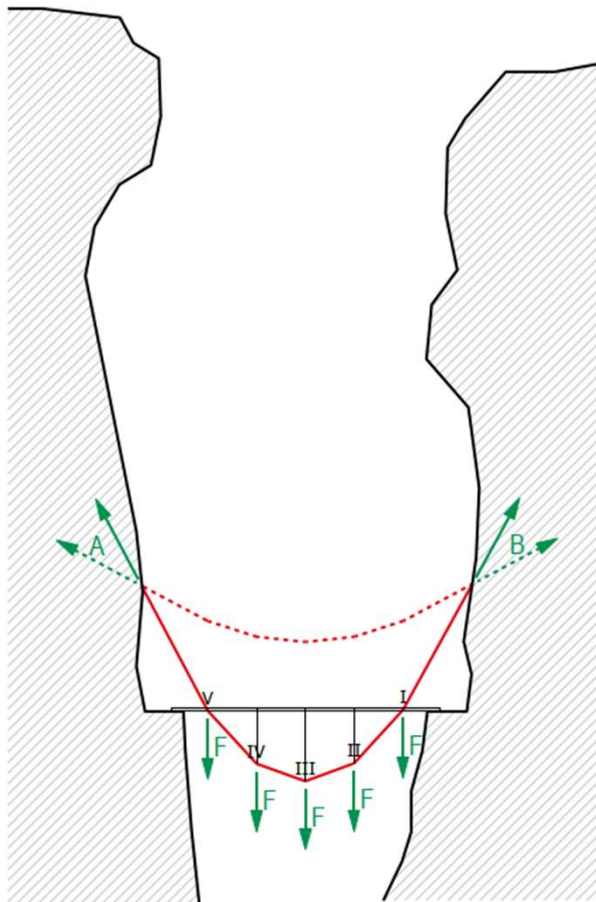
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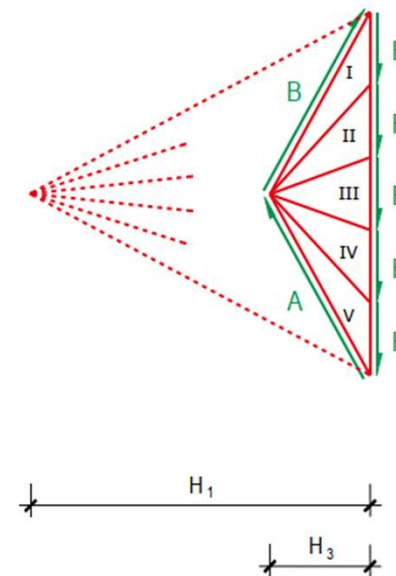
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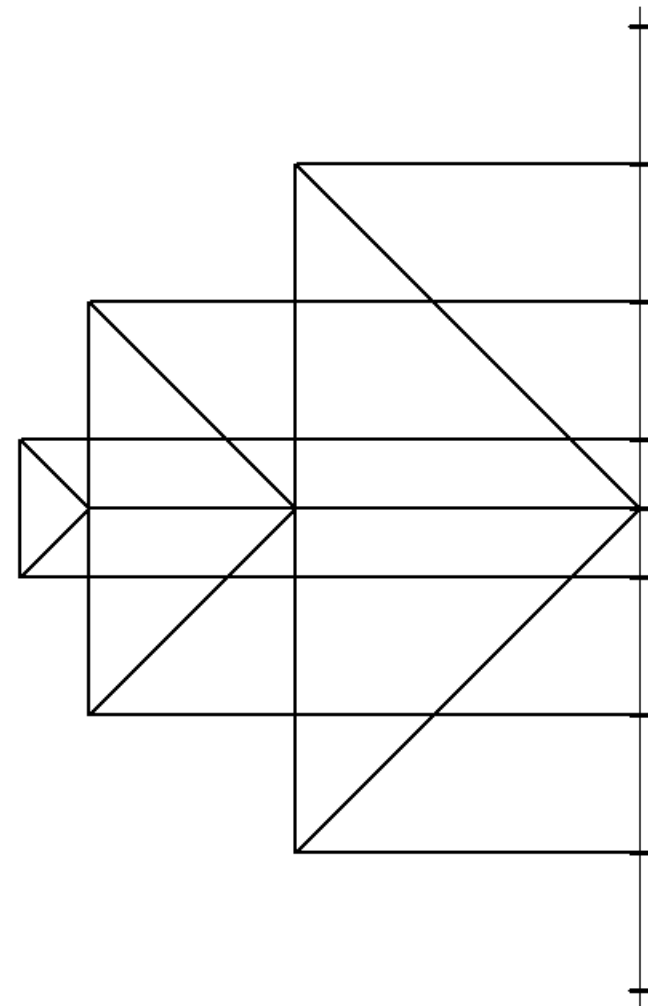
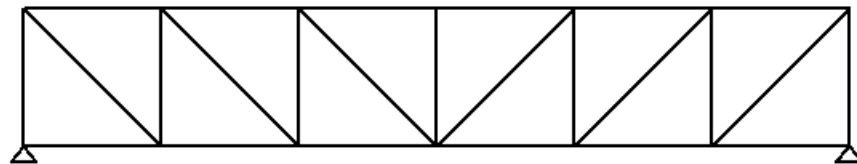


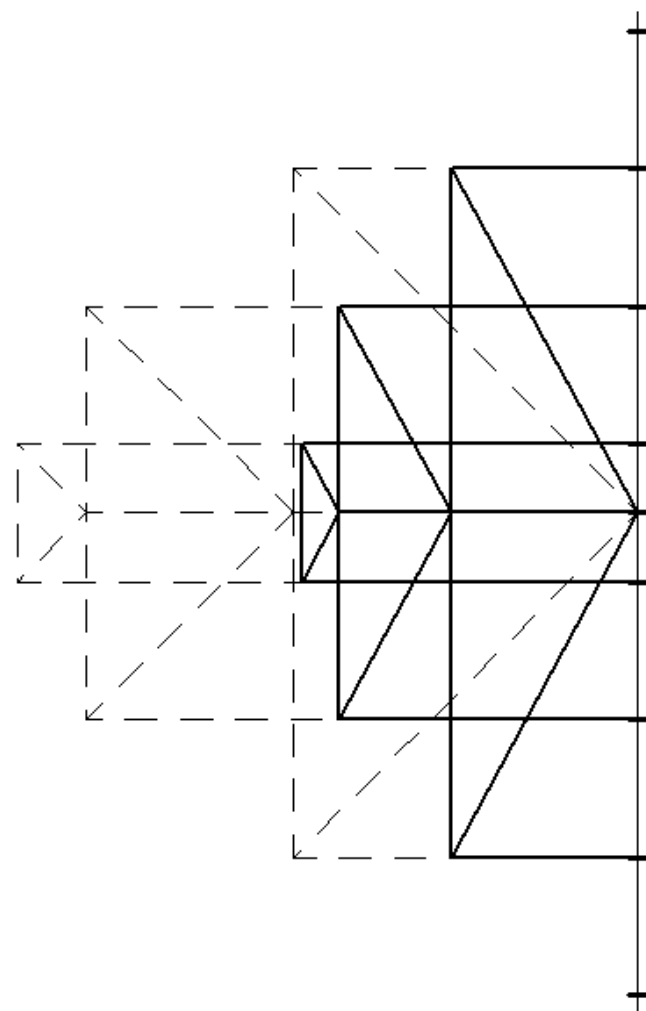
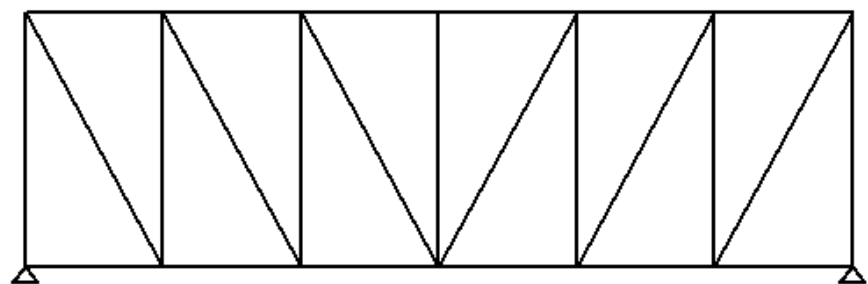
Form diagram

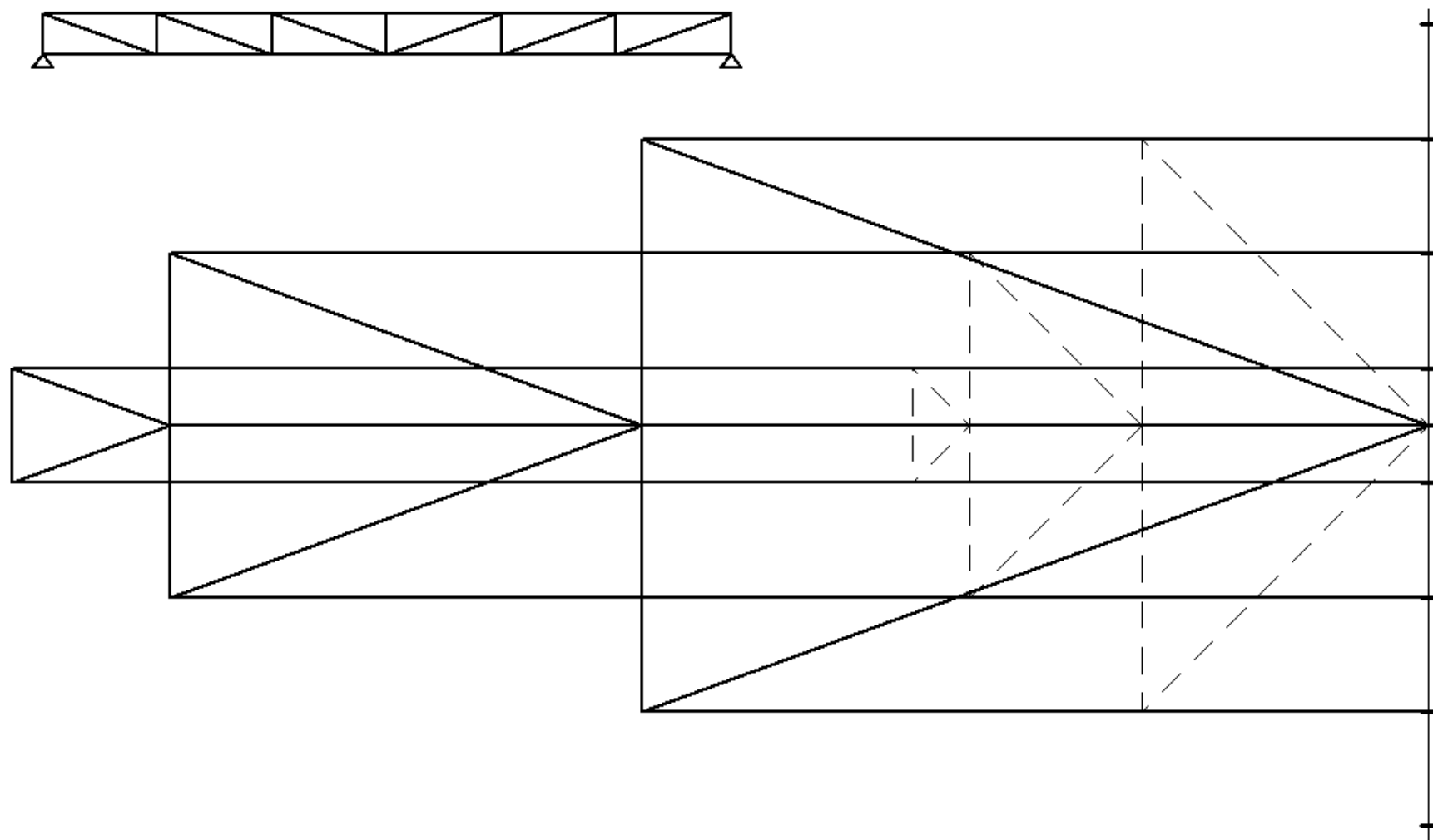


Force diagram









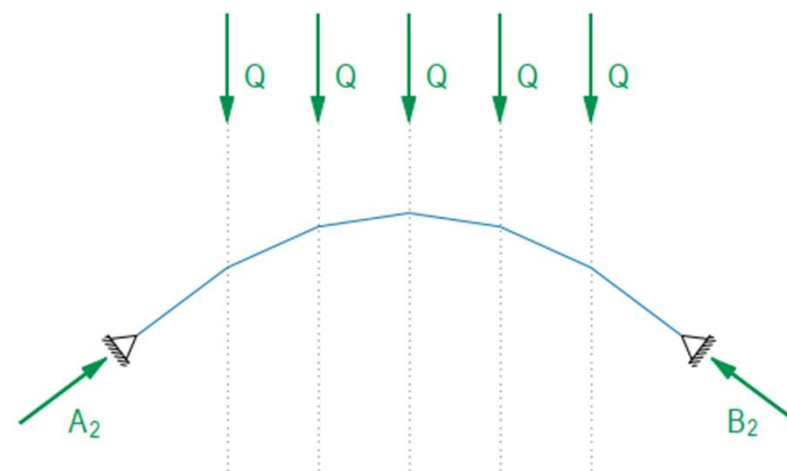
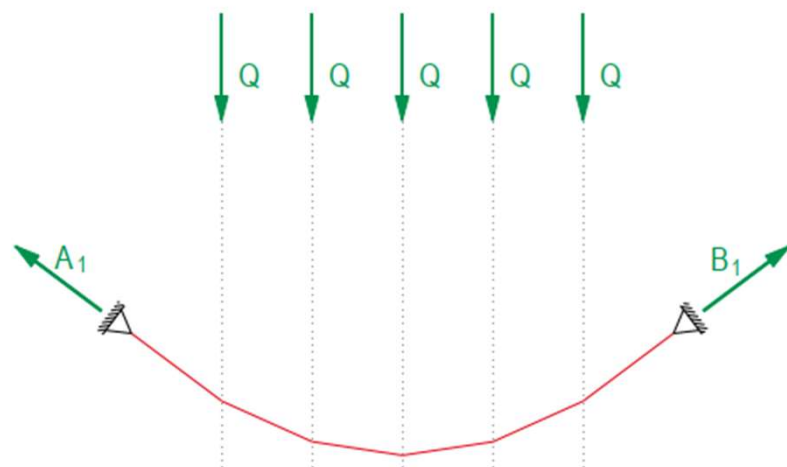
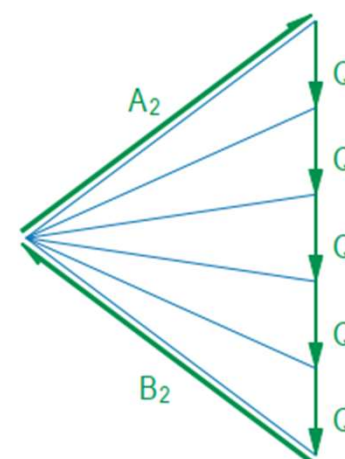
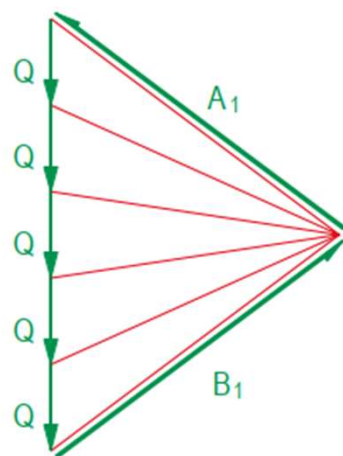
Arches

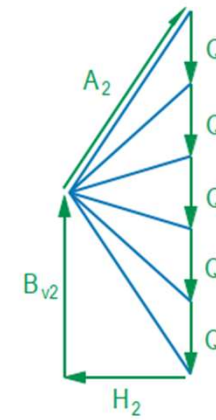
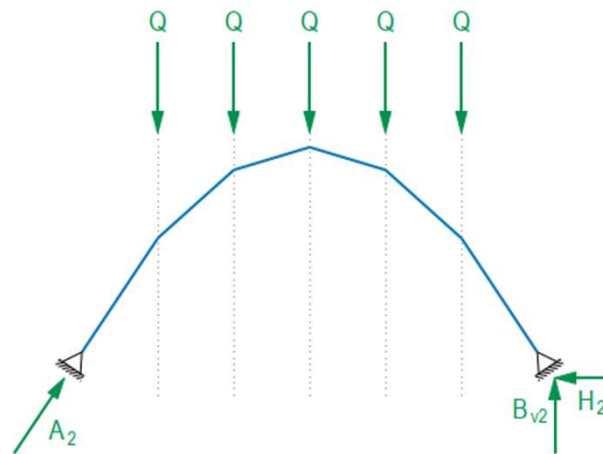
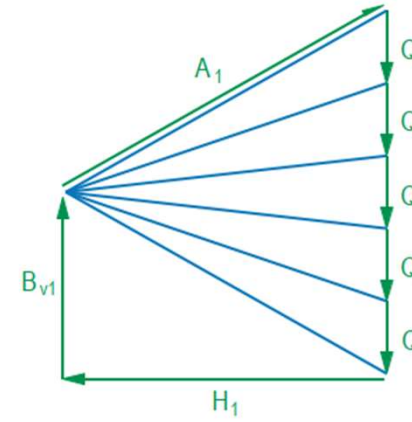
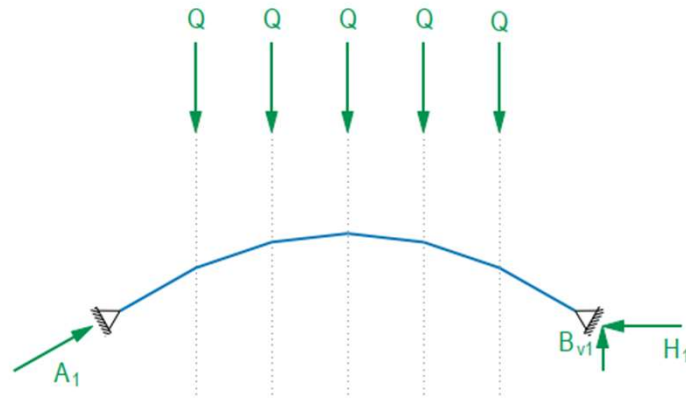


Mülheimer bridge, Cologne, 1951 | Wilhelm Riphahn & Fritz Leonardt



Valtschielbach bridge, Donath, 1925 | Robert Maillart

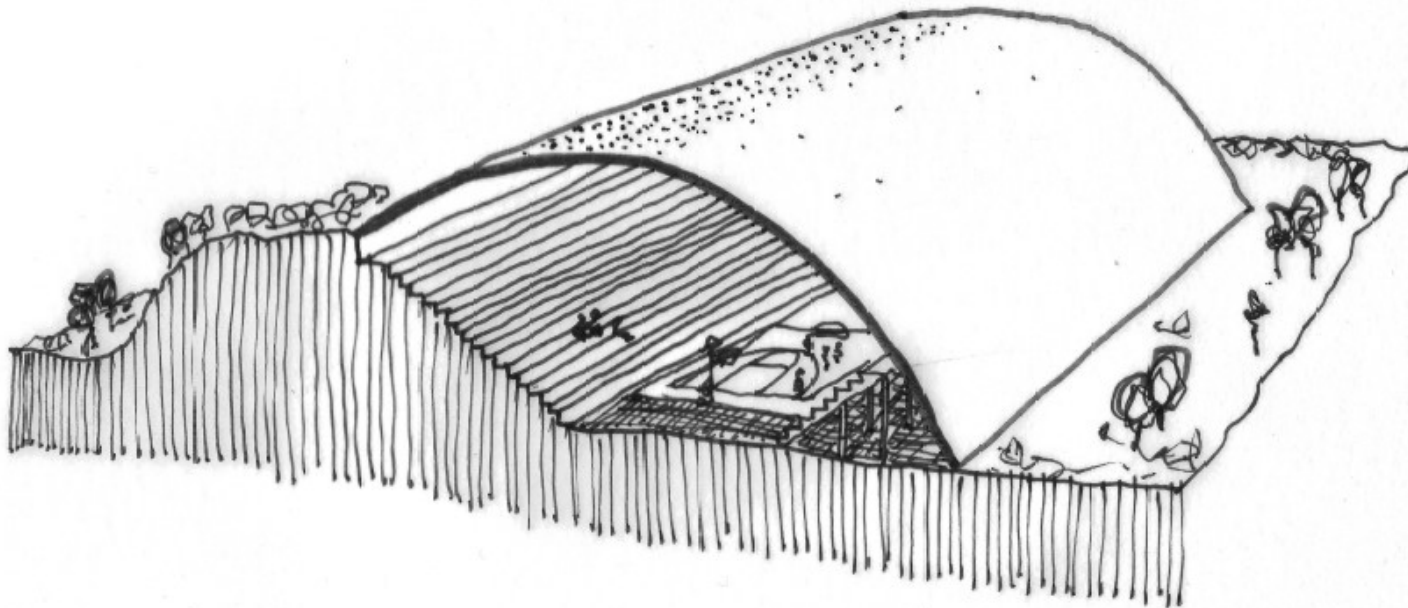




Example



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Total load ≈ 450 kN (per 1m width of roof)

SW ≈ 5 kPa

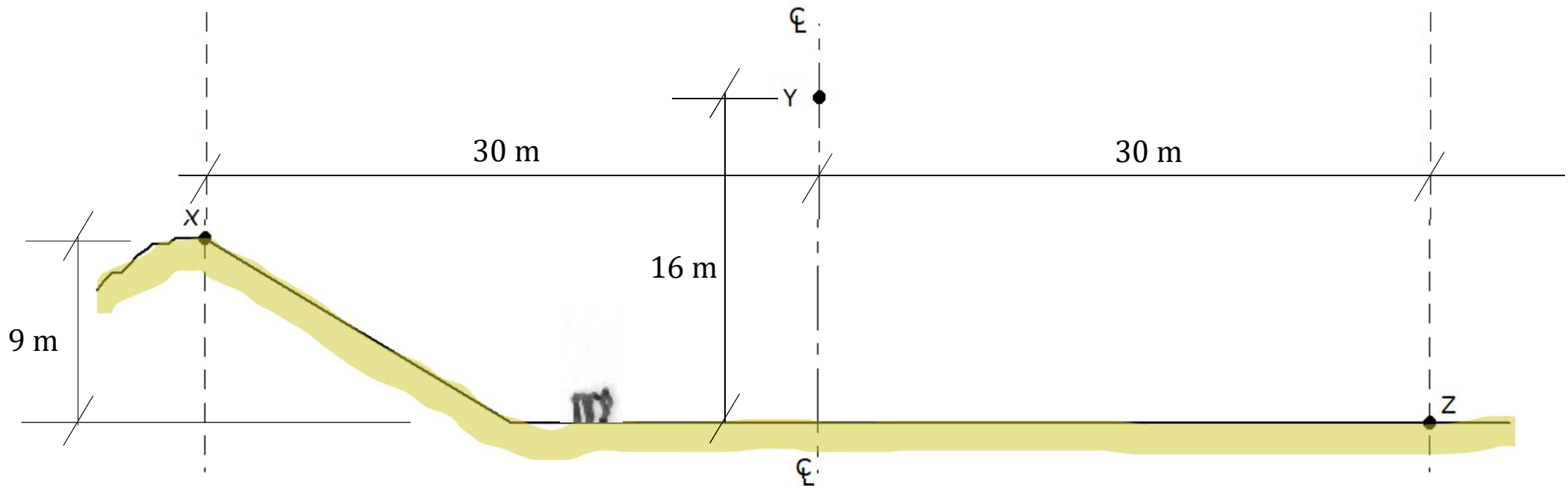
Waterproofing & insulation ≈ 0.4 kPa

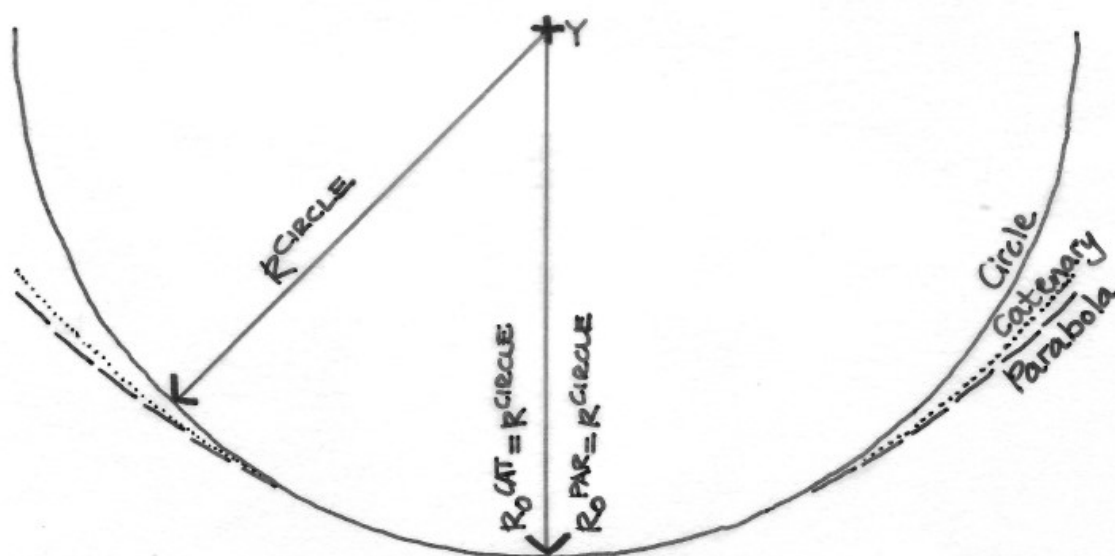
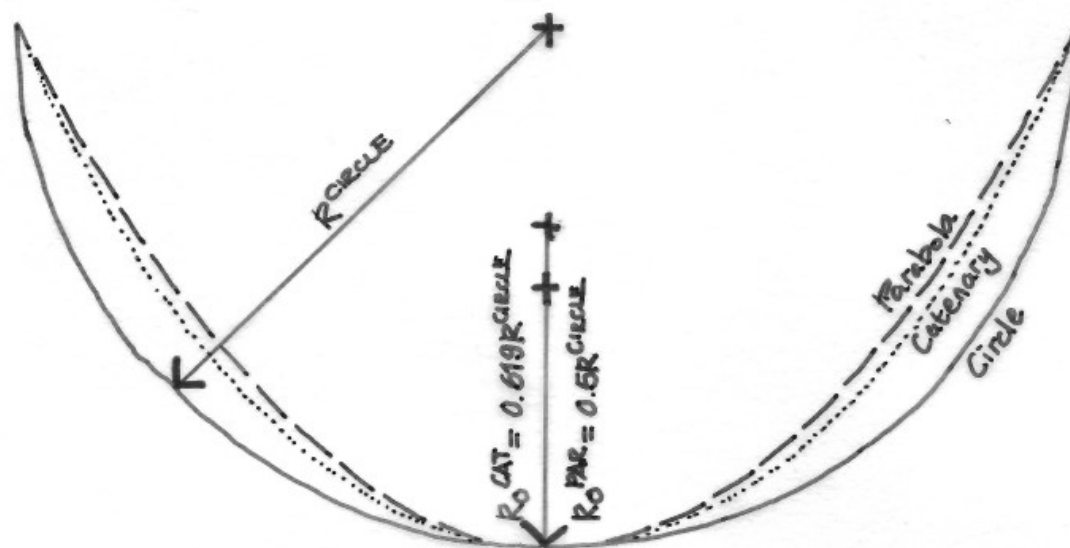
Services ≈ 0.2 kPa

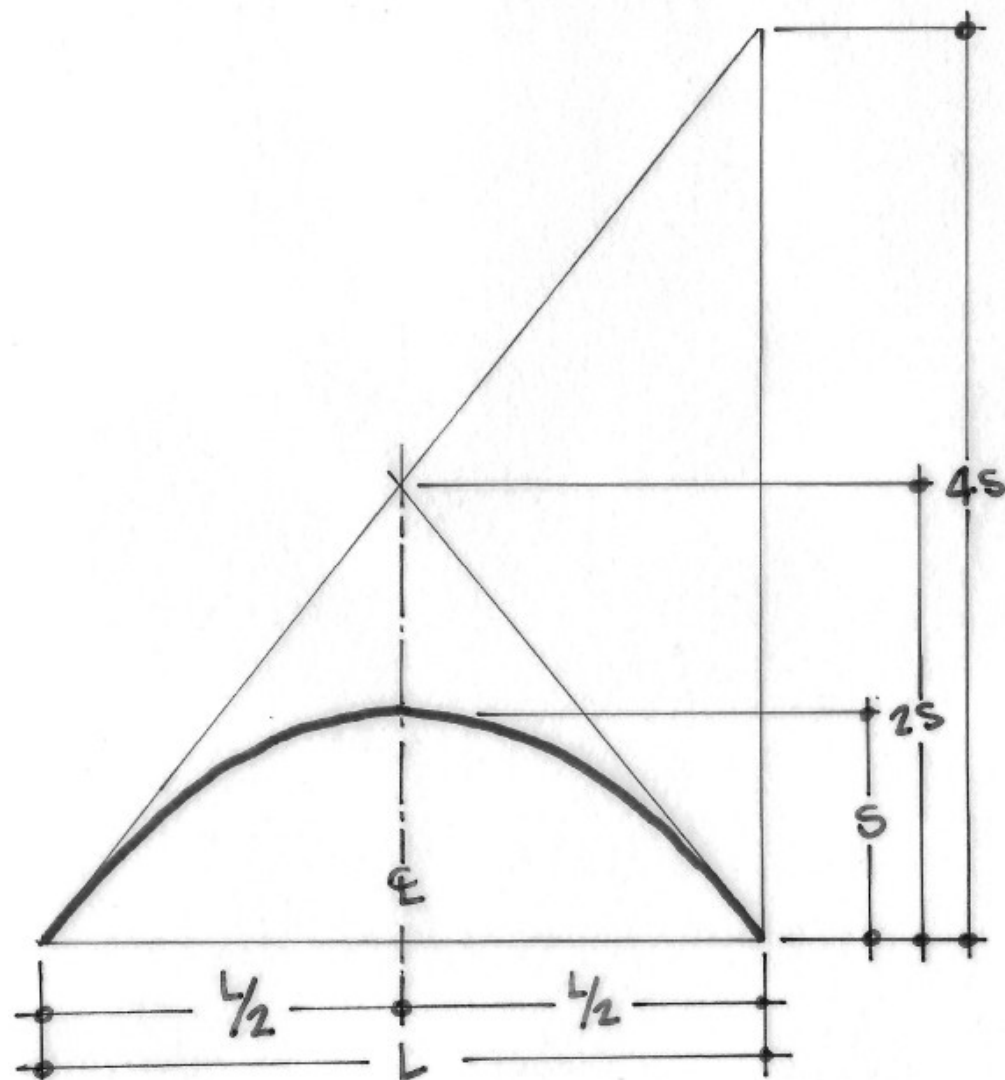
So, DL = 5.6 kPa

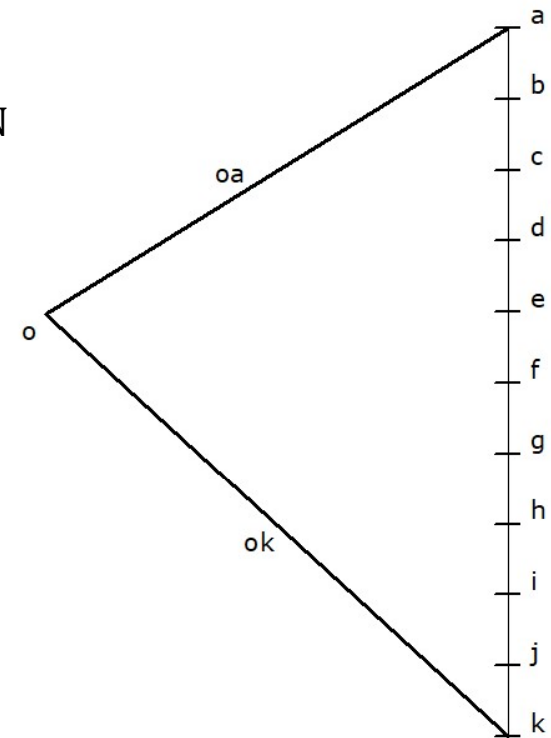
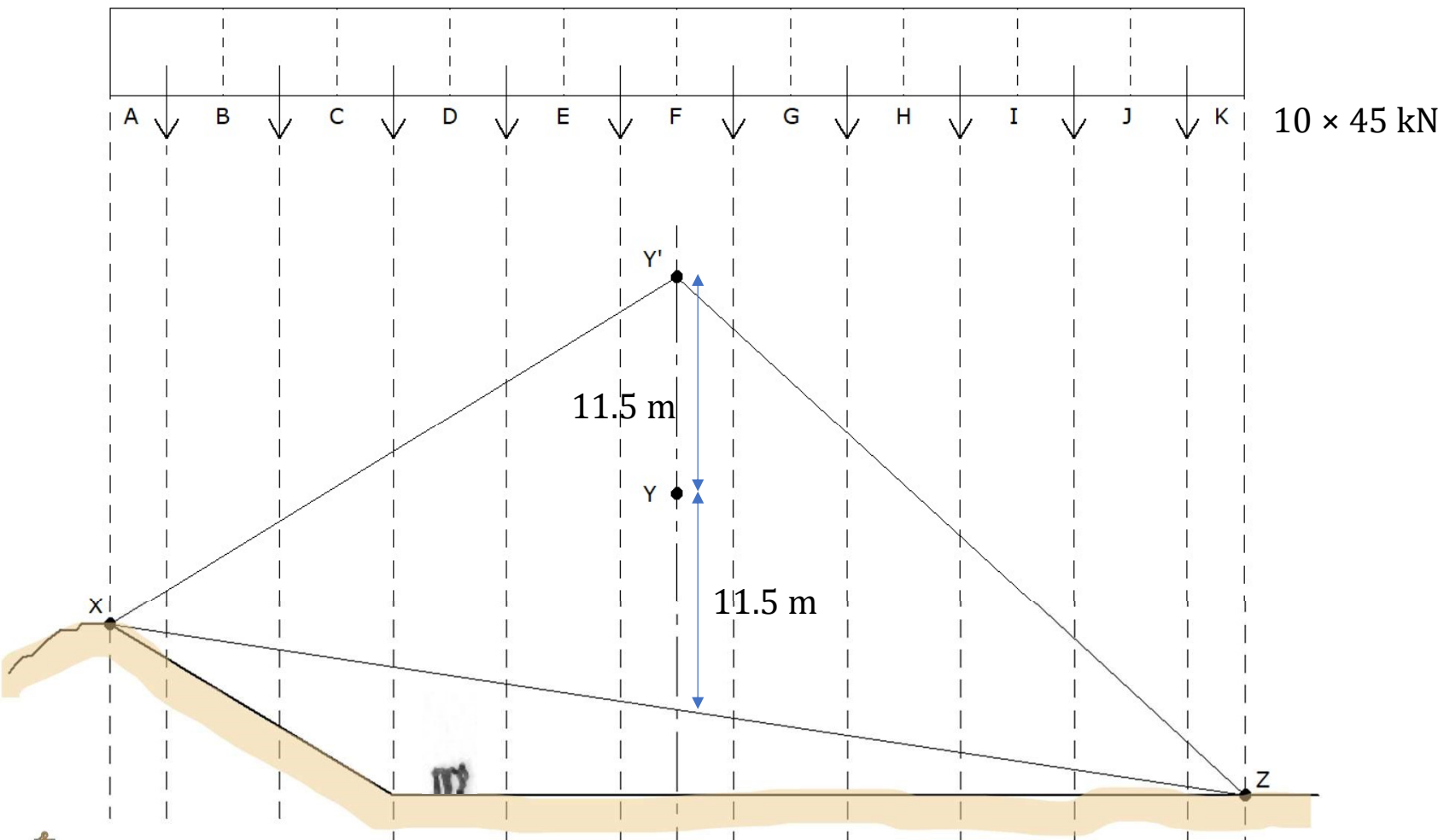
LL = 0.5 kPa

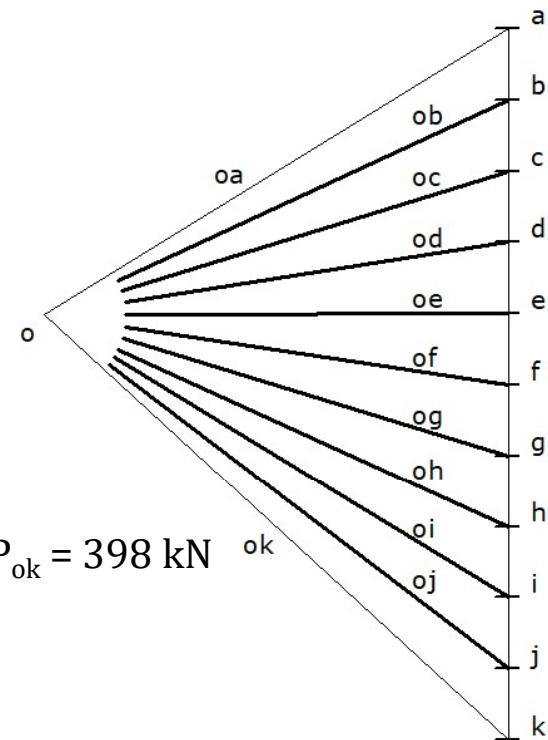
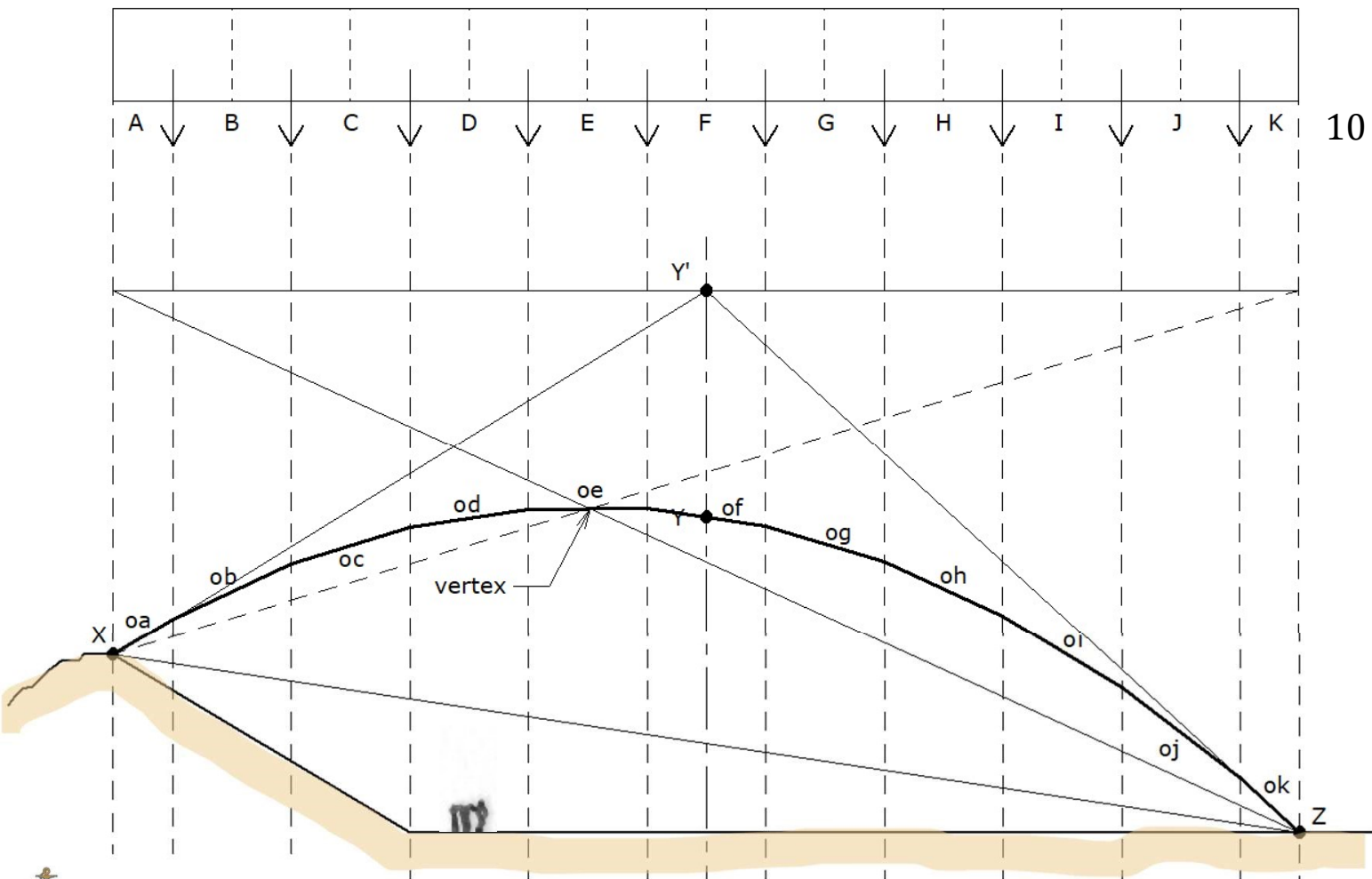
ULS = $1.2DL + 1.6LL = 7.52$ kPa











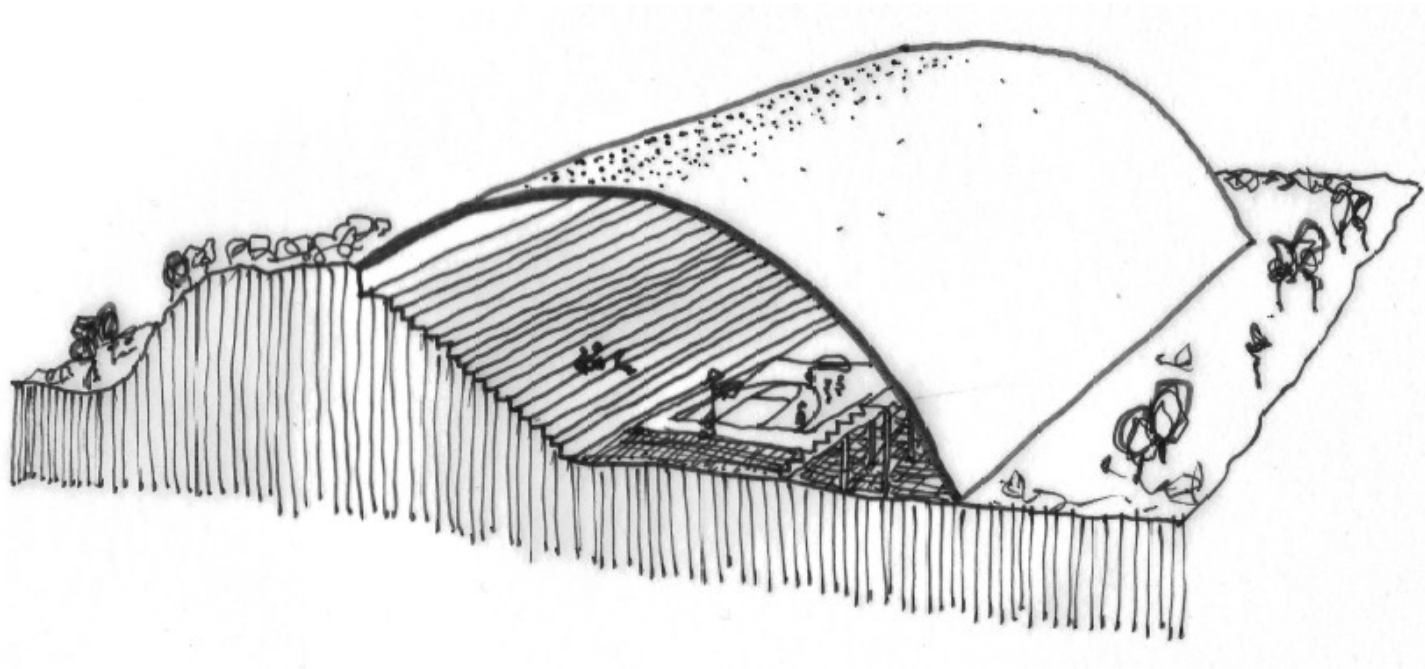
$$P_{ok} = 398 \text{ kN}$$

$$F_{\text{allow}} = P/A$$

$$4 \text{ MPa} = 398 \text{ kN} / (1000 \text{ mm} \times t)$$

$$\text{So, } t = 100 \text{ mm}$$

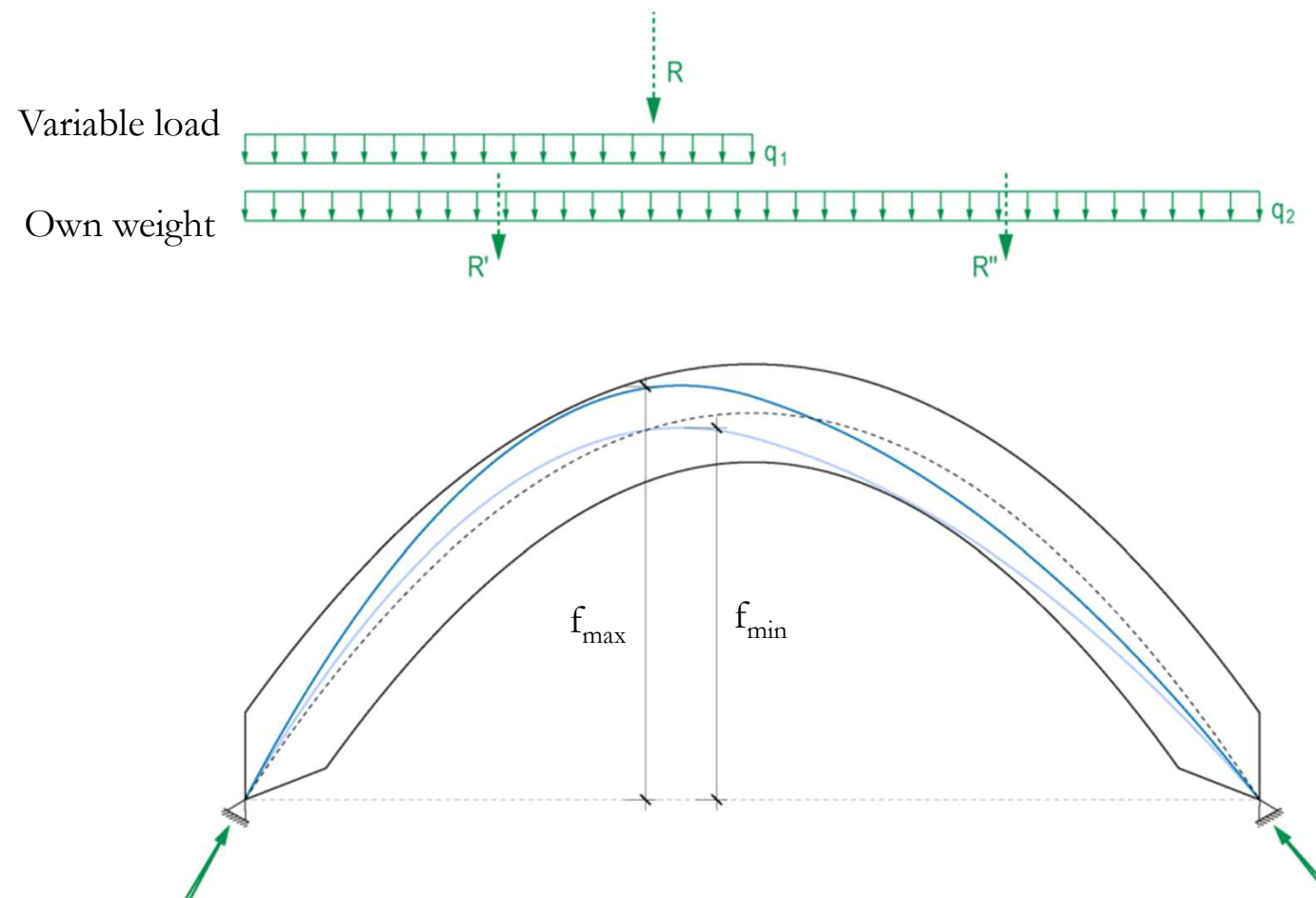




100mm??!!



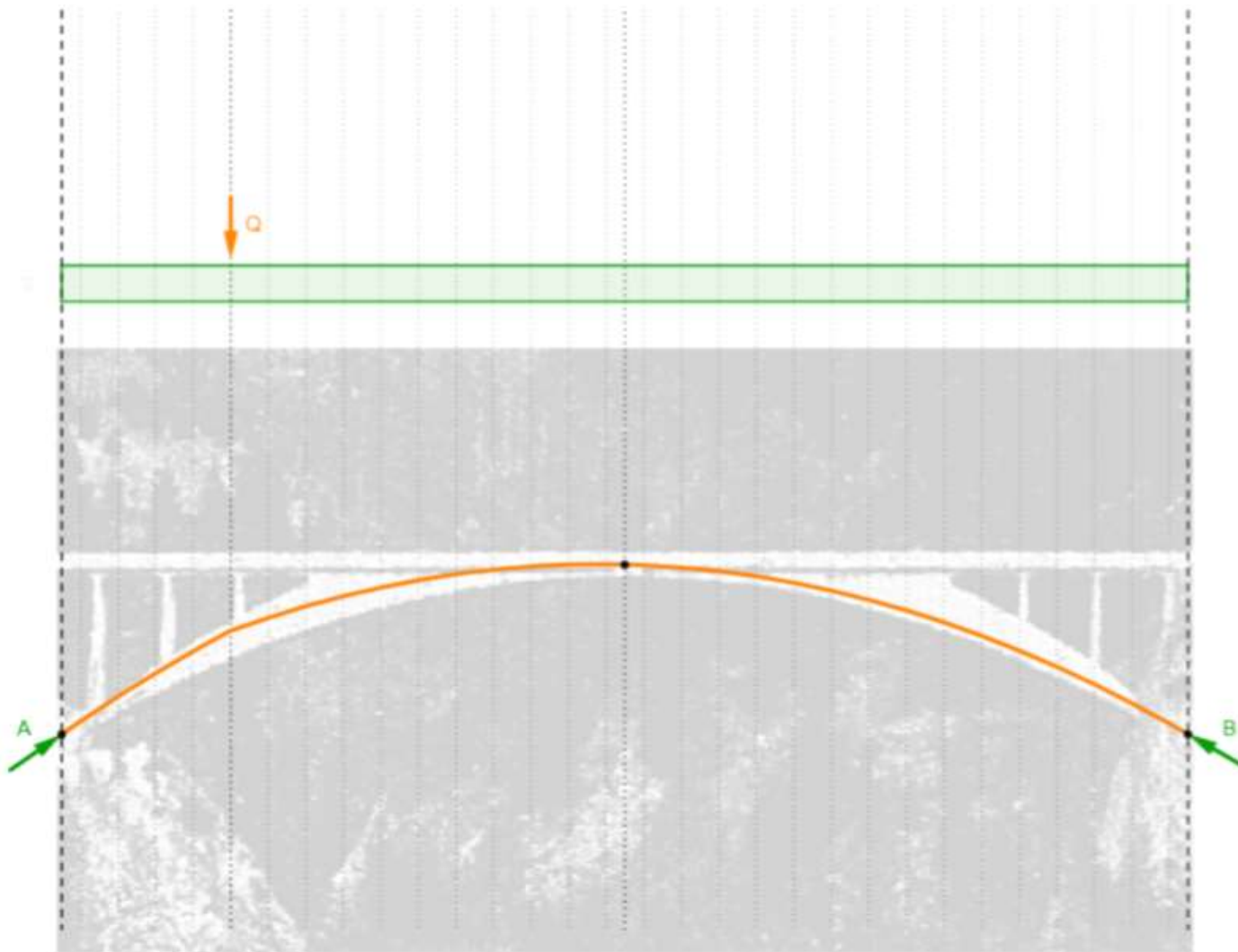
Thrust lines



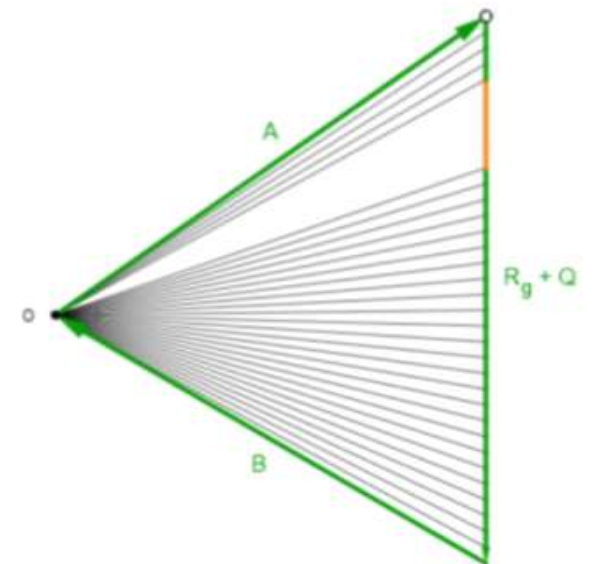


Salginatobel Bridge, Schiers, 1930 | Robert Maillart

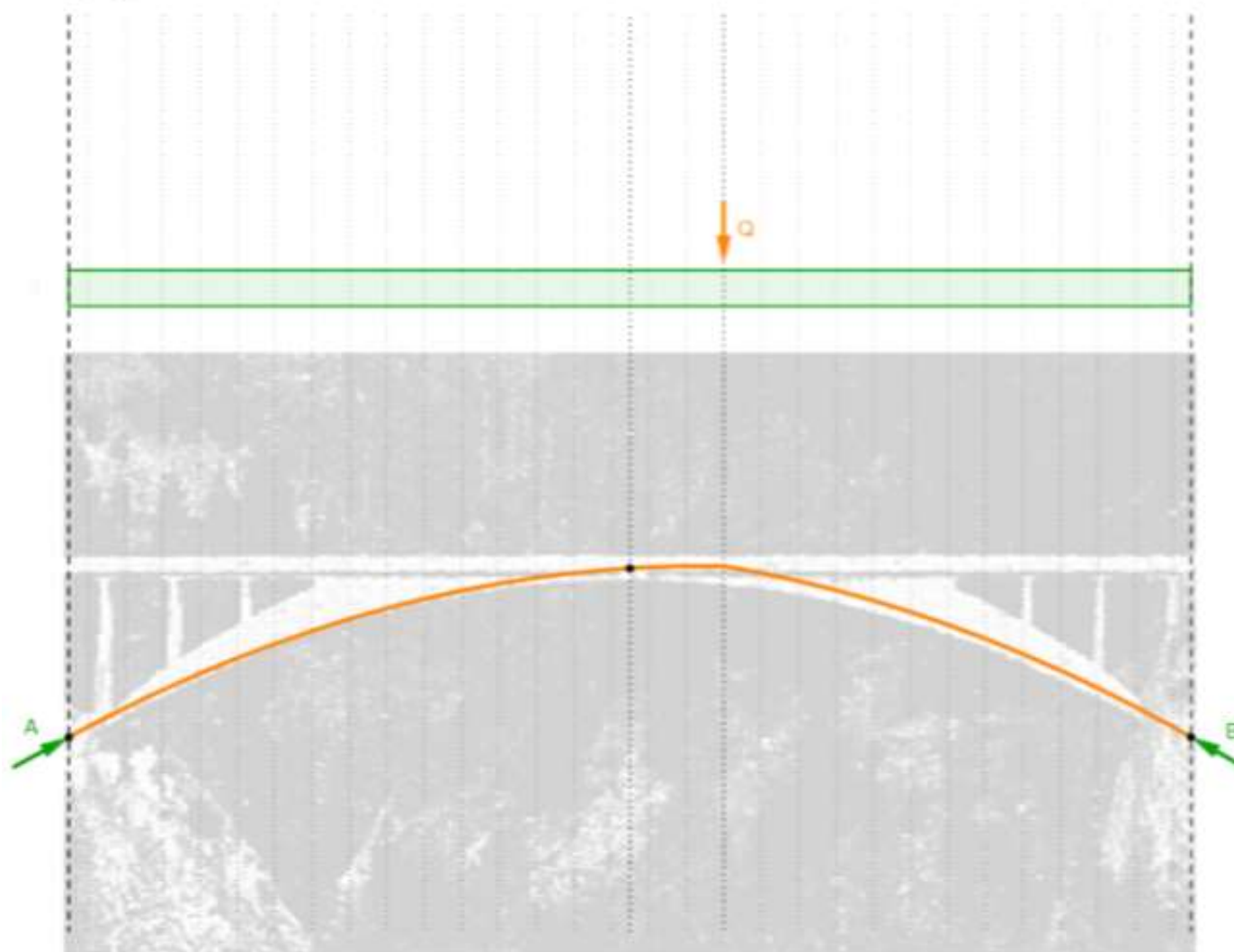
Form Diagram



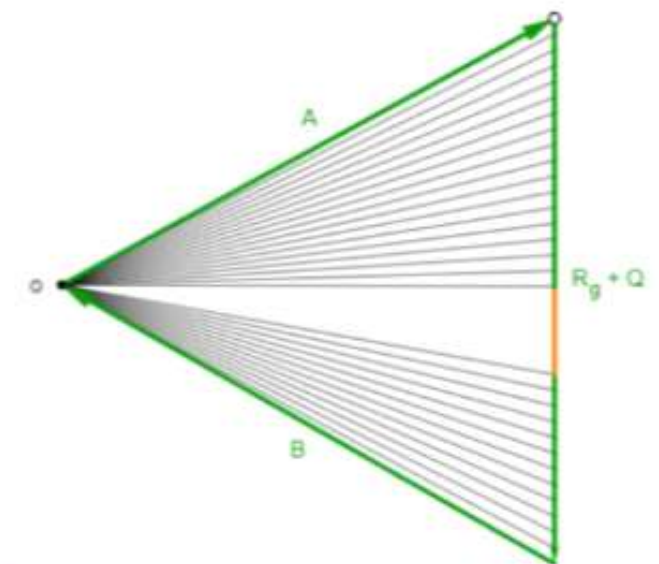
Force Diagram

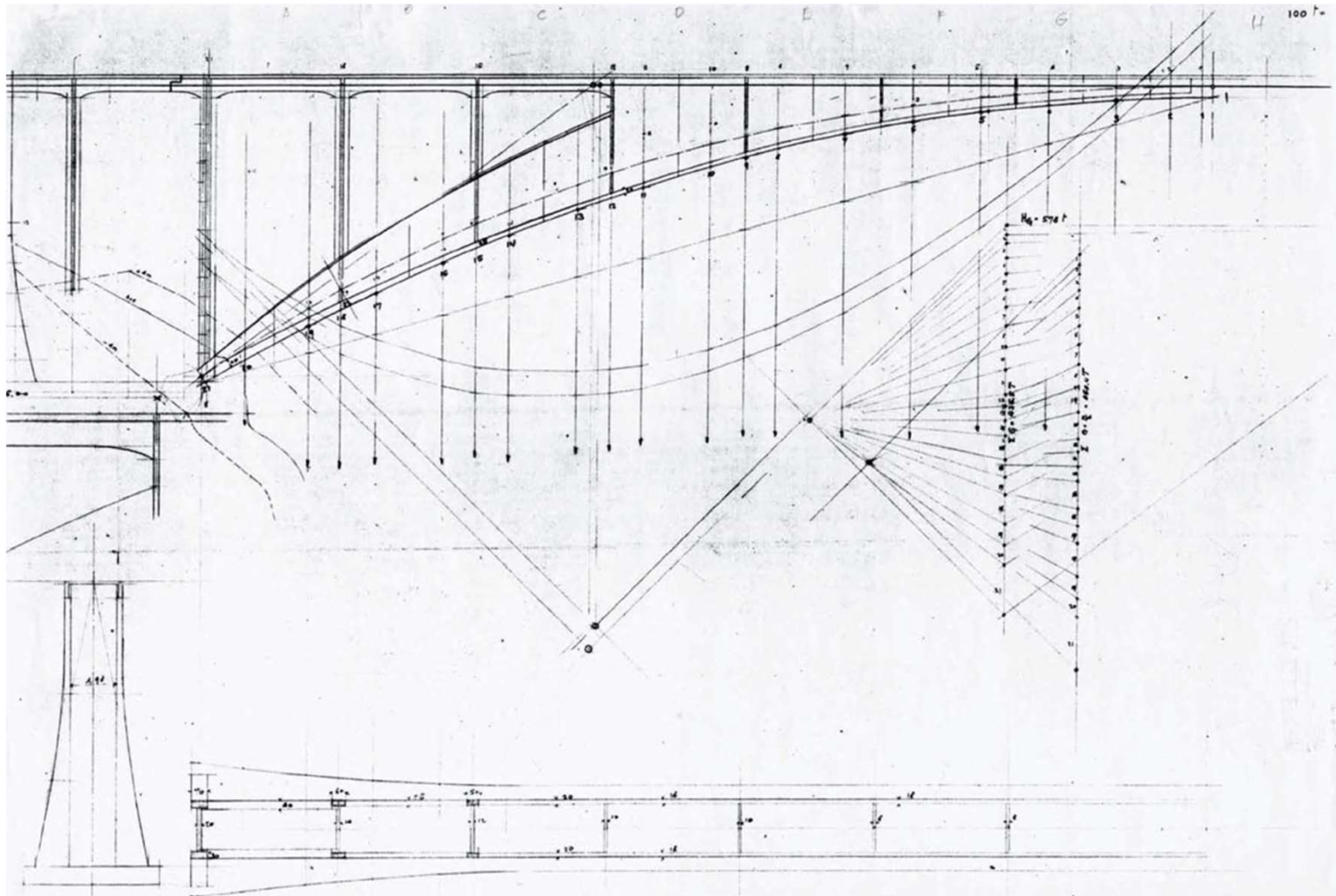


Form Diagram



Force Diagram





Salginatobel Bridge
Schiers, Switzerland
Robert Maillart (1930)



Tower Bridge
London, UK
Sir Horace Jones & Sir John Wolfe Barry (1894)

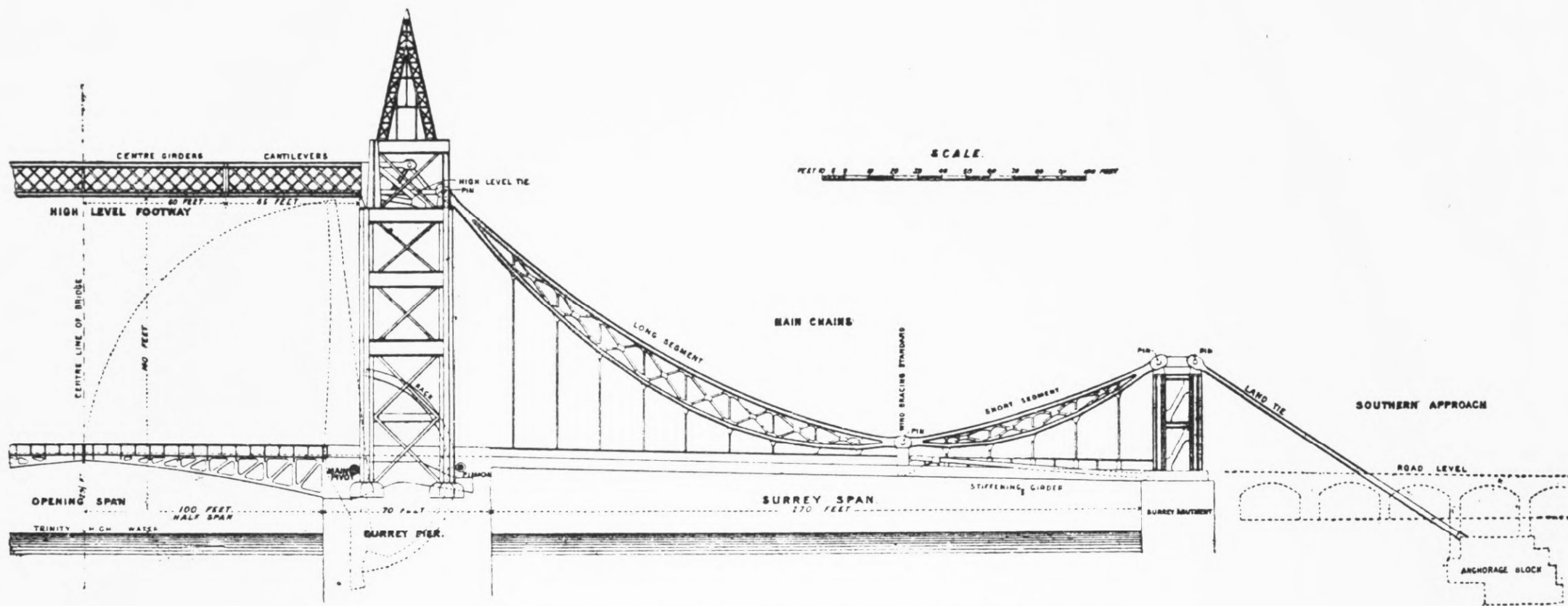


FIG. X. SKELETON HALF ELEVATION OF TOWER BRIDGE.

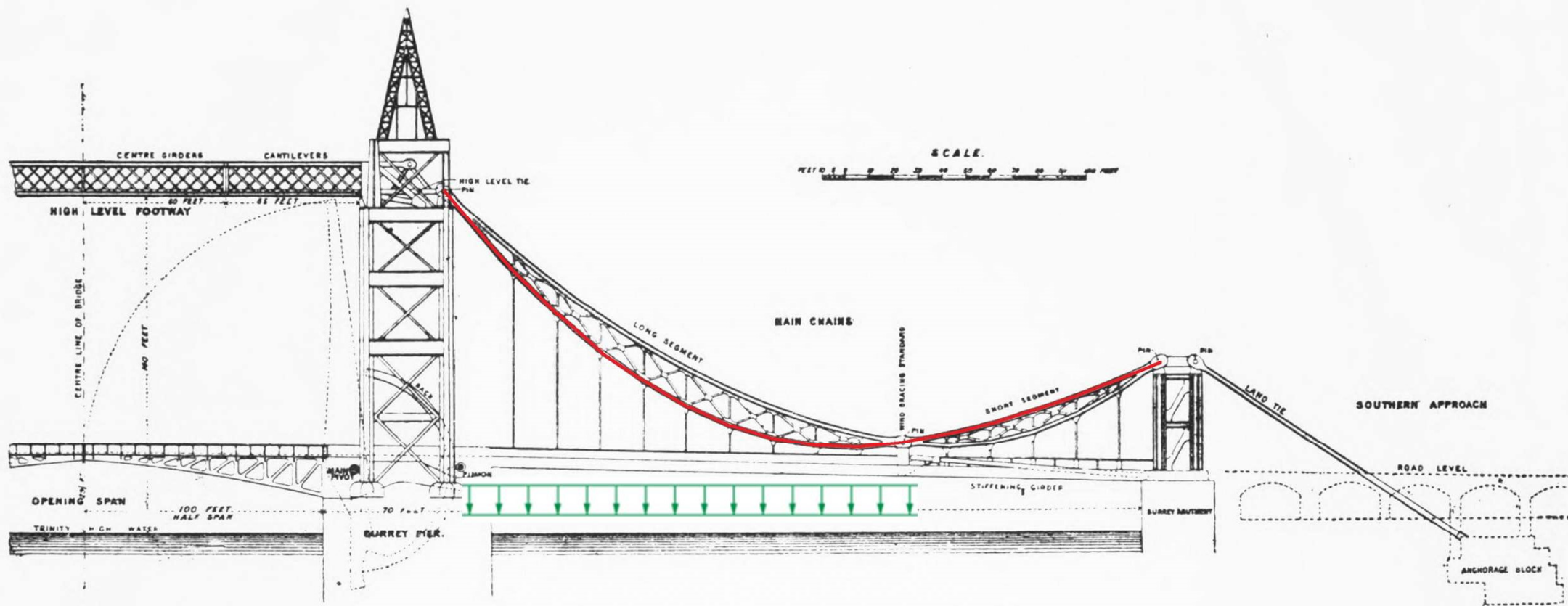


FIG. X. SKELETON HALF ELEVATION OF TOWER BRIDGE.

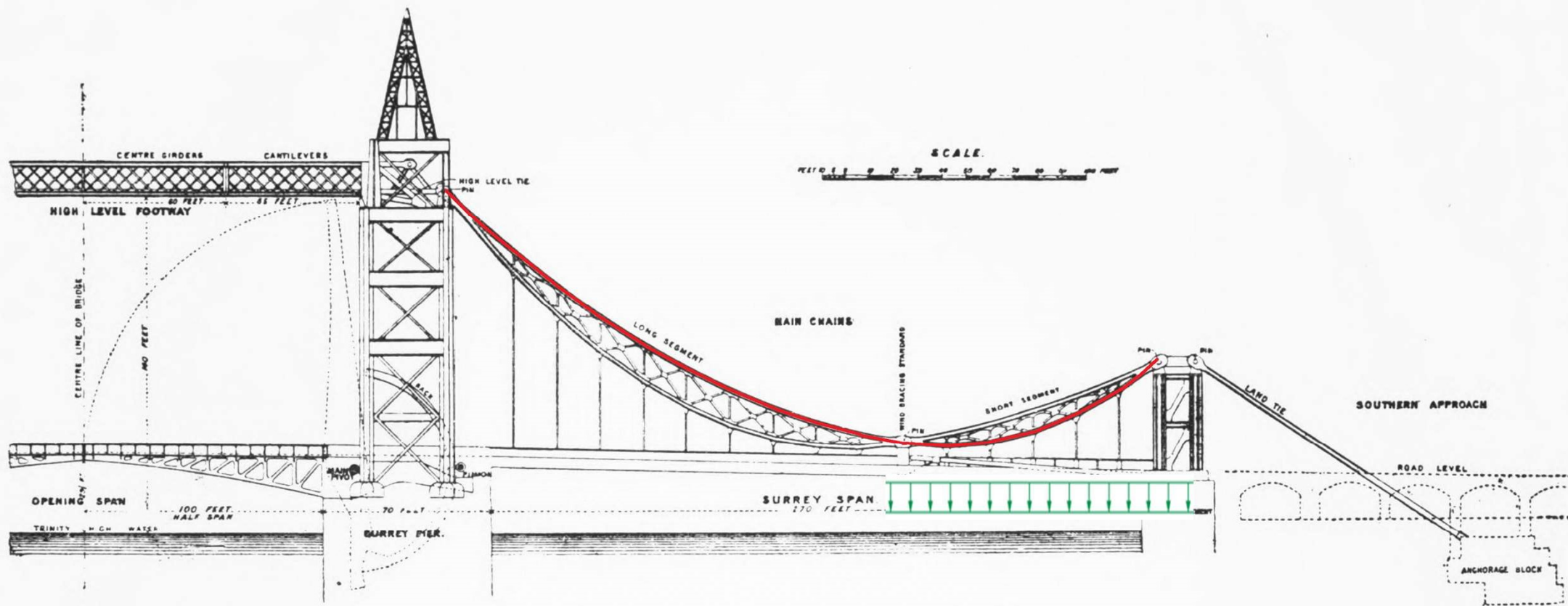
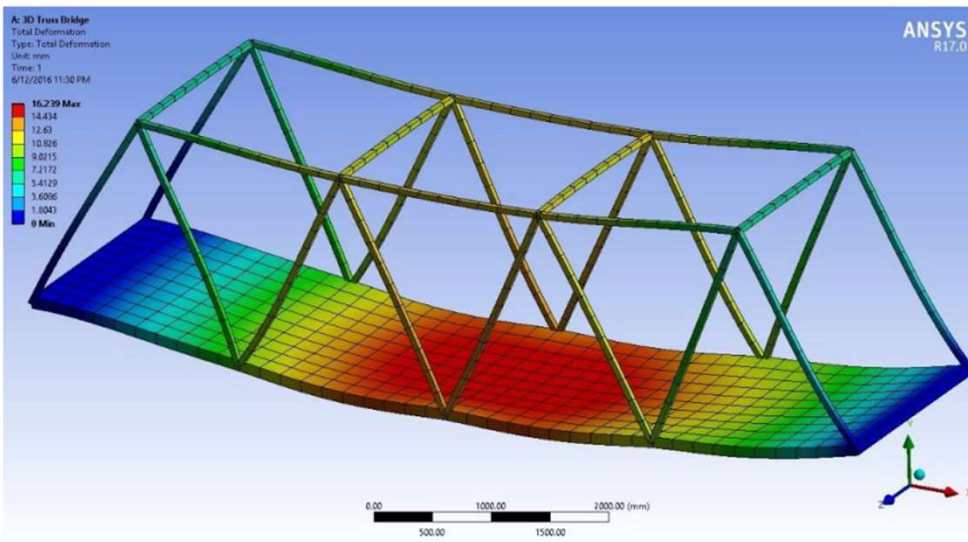
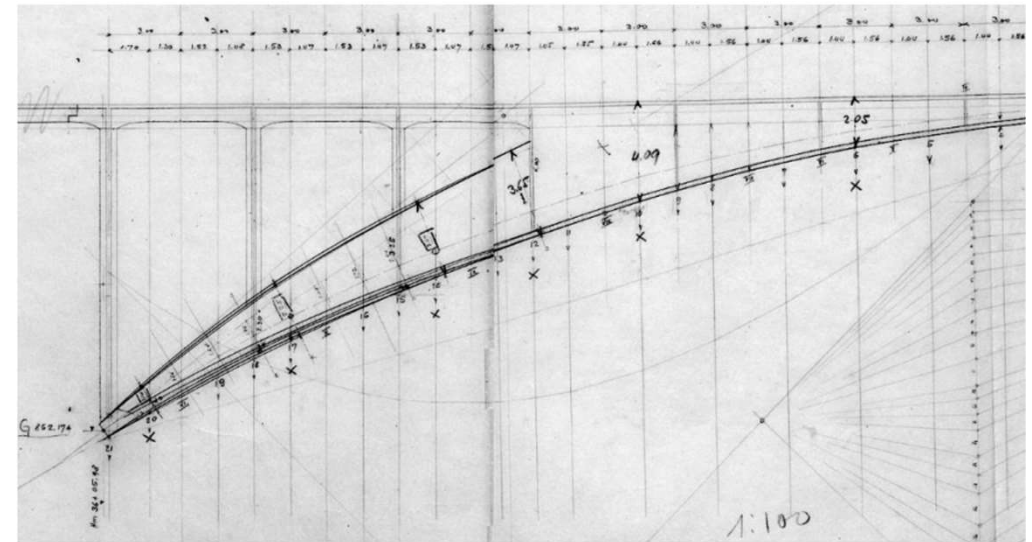


FIG. X. SKELETON HALF ELEVATION OF TOWER BRIDGE.

Concluding remarks



Structural Engineering
= analysis of forces
= reactive
= architectural monologue



Structural Design
= design with forces
= proactive
= dialog between architect and engineer

Thank you



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